PAPERS

RELATING TO

THE TRANSIT OF VENUS IN 1874,

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CHARTS.

INTRODUCTORY NOTE

The present paper was prepared under the direction of Professor J H C Coffin, Superintendent of the Nautical Almanac, and was, by request, furnished to the Commission on the Transit of Venus, to form part of the series published by it

B F SANDS,

Rear-Admiral U S N, President of the Commission.

PAPERS

RELATING TO

THE TRANSIT OF VENUS IN DECEMBER, 1874.

VI

Letter of Professor Coffin to Rear-Admiral Sands, transmitting Mr Hill's charts and tables

NAUTICAL ALMANAC OFFICE,
Washington, D C, October 1, 1872

DEAR SIR The following charts and tables, for predictions of the transit of Venus in 1874, were prepared by Mr Hill, by my direction, as a supplement to the American Ephemeris and Nautical Almanac They are so appropriate to the series of papers published by the commission appointed to make preparations for the observation of the transit that I very readily transfer them to it

I have appended some simple explanations of the charts, and directions for their use They are designed particularly for navigators in the Pacific and Indian Oceans, who, accustomed to note carefully the times of their observations, may desire to observe this transit, but may not be able to comprehend Mr Hill's paper

I am, very respectfully, your obedient servant,

J H C COFFIN,
Professor of Mathematics, U S N, Superintendent

Rear-Admiral B F SANDS, U S N,
Superintendent of U S Naval Observatory

VII

Charts and tables for facilitating predictions of the several phases of the transit of Venus in December, 1874, prepared by Mr George W Hill, assistant in the Nautical Almanac Office

NYACK TURNPIKE, NEW YORK, *June* 29, 1872

DEAR SIR I respectfully present the following report in relation to the transit of Venus in 1874

All the constants and elements which have been used in the computations on the transit are given below The quantities having no terms multiplied by t are either constant or may be regarded as such for the duration of the transit, and the quantities which vary may be regarded as varying uniformly The unit of t is an hour

Epoch: 1874, *December* 8^d 11^h, *Washington mean time.*

VENUS.

Orbit longitude, referred to the mean equinox of date	$76° 58' 12''.84 + 242''.332\,t$
Longitude of the ascending node	$75° 33' 24''.1$
Log sine of inclination	8.7722486
Periodic perturbations of the latitude	$+0''.11$
Log radius-vector	$9.8575310 - 27.6\,t$
Semi-diameter at mean distance	$8''.546$

THE SUN.

True longitude, referred to the mean equinox of date	$256° 58' 41''.62 + 152''.532\,t$
Latitude	$-0''.41$
Log radius-vector	$9.9932845 - 21.3\,t$
Semi-diameter at mean distance	$959''.788$
True obliquity of the ecliptic	$23° 27' 27''.67$
Equation of the equinoxes in longitude	$-7''.42$
Sidereal time, at Washington, in arc	$62° 44' 9''.6 + (15° 2' 27''.84)t$
Constant of solar parallax	$8''.848$
Constant of aberration	$20''.4451$
Eccentricity of the earth's meridians	0.0816967
Horizontal refraction	$35'$

The elements of the heliocentric position of Venus are from the new Tables of Venus,* and may be readily deduced from the first example given in pages 16–19 of the introduction.

The apparent position of the sun which results from the above elements coincides with that derived from the tables of Hansen and Olufsen, but the true longitude is $0''.19$ greater, owing to the adoption of Struve's value of the constant of aberration, $20''.445$, instead of the value $20''.255$.

The value of the sun's semi-diameter is adopted from Bessel. (See *Astronomische Nachrichten*, No. 228, and *Astronomische Untersuchungen*, Vol. II, p. 114.) This value is used in the computation of eclipses for the American Ephemeris. Hansen has also used it in his disquisition on the transit of Venus. In the British Nautical Almanac the value $961''.82$ is used, and is the same as that given for the reduction of meridian-observations of the sun. Leverrier states (*Annales*, Vol. VI, p. 40) that the value, deduced from the previous transits of Venus, is $958''.424$. Hence it is probable that predictions from the elements of the British Nautical Almanac will be found to be considerably in error from this cause.

From the data given above are obtained the following hourly ephemerides. For the sake of completeness they are expressed in terms of longitude and latitude, as well as in right ascension and declination.

* Tables of Venus, prepared for the use of the American Ephemeris and Nautical Almanac, by George W. Hill. Washington, 1872.

VENUS

Wash M T	$\alpha =$ App R A	$\delta =$ App dec	App geocentric longitude	App geocentric latitude	Log r = log distance from the earth
1874	° ′ ″	° ′ ″	° ′ ″	′ ″	
Dec 8d 8h	255 58 56 03	− 22 38 9 96	257 4 53 34	+ 11 40 84	9 4221505
9	57 21 96	37 22 29	3 22 30	12 19 91	482
10	55 47 90	36 34 60	1 51 27	12 58 99	467
11	54 13 86	35 46 90	257 0 20 24	13 38 06	460
12	52 39 84	34 59 18	256 58 49 21	14 17 13	461
13	51 5 83	34 11 44	57 18 18	14 56 21	470
14	255 49 31 85	− 22 33 23 67	256 55 47 16	+ 15 35 28	9 4221488

THE SUN

Wash M T	$\alpha' =$ App R A	$\delta' =$ App dec	App longitude	App latitude	Log r' = log distance from the earth
1874	° ′ ″	° ′ ″	° ′ ″	″	
Dec 8d 8h	255 42 16 80	− 22 48 24 39	256 50 35 86	− 0 41	9 9932909
9	45 1 47	48 39 36	53 8 39	0 41	888
10	47 46 15	48 54 28	55 40 93	0 41	867
11	50 30 84	49 9 15	256 58 13 44	0 41	845
12	53 15 54	49 23 98	257 0 45 99	0 41	824
13	56 0 25	49 38 77	3 18 52	0 41	802
14	255 58 44 98	− 22 49 53 51	257 5 51 05	− 0 41	9 9932781

From these quantities the position of the center of the sun, as seen from the center of Venus, is derived

Wash M T	$a =$ R A	$d =$ dec	Log G = log distance
1874	° ′ ″	° ′ ″	
Dec 8d 8h	255 36 9 50	− 22 52 9 48	9 8575394
11	255 49 8 82	54 3 58	309
14	256 2 8 52	55 56 63	227

In the next place are obtained the following quantities, which are designated by the eclipse-notation* of Chauvenet's Spherical and Practical Astronomy, which, for the

* The plane of reference passes through the center of the earth perpendicular to the axis of the enveloping cones, a and d are the right ascension and declination of the vanishing point of the axis, μ_1, the hour-angle of that point at the first meridian, G, the distance of the sun and planet, x, y, the co-ordinates of the axis in the plane of reference, y being taken positive toward the north, x positive toward that point whose right ascension is $90° + a$, $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are the hourly changes of x and y, f is the angle of the cone, l, the radius of the cone in the plane of reference, $i = \tan f$.

most part, is identical with that of Bessel's *Analyse der Finsternisse.* It must be remembered that Venus here takes the place of the moon.

Wash. M. T.	x	$\frac{dx}{dt}$	y	$\frac{dy}{dt}$	μ_1
1874.					° ′ ″
Dec. 8d 8h	+ 37.6744	− 9.74895	+ 25.0318	+ 2.59020	122 0 36.6
11	+ 8.4134	9.75838	32.7602	2.56207	166 55 0.8
14	− 20.8759	− 9.76782	+ 40.4042	+ 2.53393	211 49 24.6

Wash. M. T.	Exterior contacts.				Interior contacts.			
	f	l	log l	log i	f	l	log l	log i
1874.	′ ″				′ ″			
Dec. 8d 8h	22 24.272	41.1254	1.614110	7.8141	22 0.545	38.4845	1.585286	7.8063
11	.299	62	19	41	.570	54	296	63
14	.324	68	124	41	.595	59	301	63

CURVES REPRESENTED ON THE CHARTS.

Having now the necessary data, I proceed to explain the computations which have been made for the purpose of drawing the charts. These charts are designed to give the principal circumstances attending each of the four contacts at any point of the earth's surface where it is visible. These circumstances may be taken to be the time at which the contact occurs, and the position of the point of contact on the sun's limb. Hence, two classes of curves have been plotted on the charts—first, *curves upon which contact occurs at the same instant;* and, secondly, *curves upon which contact takes place at the same point on the sun's limb.* These curves are evidently limited, in both directions, by the curve upon which contact takes place in the horizon. The readiest method of drawing them will be to compute a sufficient number of positions conveniently distributed on these curves, and through these positions, plotted on the chart, draw the curves.

As convenient formulæ for the purpose are not found in the treatises on practical astronomy, I will develop them here.

It will be amply sufficient to determine the position of these curves to within a minute of arc. Hence, as the horizontal parallax of Venus is only 33″, the effect of parallax on the right ascension and declination of the point of contact may be neglected. Then the position of this point can be found by the equations,

$$a' = \alpha \pm \frac{s}{s' \pm s}(\alpha' - \alpha)$$

$$d' = \delta \pm \frac{s}{s' \pm s}(\delta' - \delta)$$

the upper sign being used for the exterior contacts, and the lower for the interior With sufficient approximation, these equations may be written

$$a' = \alpha \pm \frac{1}{30} (\alpha' - \alpha)$$

$$d' = \delta \pm \frac{1}{30} (\delta' - \delta)$$

The exterior contacts last about 21 minutes on the earth's surface, and the interior contacts about 25 minutes The quantities a' and d' vary so slowly that they may be computed for the middle of the duration of each contact on the earth's surface, and then supposed constant for this duration In this way the following values have been obtained

	Wash M T	a'	d'
	h m	° ′	° ′
For exterior contact at ingress	8 40	255 57	− 22 38
For interior contact at ingress	9 10	255 57	22 37
For interior contact at egress	12 48	255 51	22 34
For exterior contact at egress	13 18	255 51	− 22 34

The investigation to be made is conveniently divided into two problems

PROBLEM I—*To find the point of the earth's surface at which contact takes place at a given time and at a given altitude*

Let

$\omega =$ the longitude of the required point west from the first meridian,
$\varphi =$ its latitude,
$\mu =$ the sidereal time at the first meridian,
$h =$ the given altitude,
$\theta =$ the parallactic angle at the point of contact,
$\vartheta' = \mu - a' - \omega =$ the hour-angle of the point of contact

The general formulæ of spherical trigonometry, applied to the triangle formed by the zenith, the pole, and the point of contact, give these equations

$$\cos\varphi \sin\vartheta' = \cos h \sin\theta$$
$$\cos\varphi \cos\vartheta' = \cos d' \sin h - \sin d' \cos h \cos\theta$$
$$\sin\varphi = \sin d' \sin h + \cos d' \cos h \cos\theta$$

As soon as θ is known, these three equations, together with the equation,

$$\omega = \mu - a' - \vartheta' = \mu'_1 - \vartheta'$$

give the position of the required point To obtain θ, resort must be had to the equation defining the condition of contact, viz

$$(l - i\zeta)^2 = (x - \xi)^2 + (y - \eta)^2$$
$$= x^2 + y^2 - 2(x\xi + y\eta) + \rho^2 - \zeta^2$$

In place of x and y make the usual substitutions,

$$x = m \sin M$$
$$y = m \cos M$$

then,

$$\xi \sin M + \eta \cos M = \frac{m^2 - (l - i\zeta)^2 + \rho^2 - \zeta^2}{2m}$$

The numerical value of each member of this equation is always less than unity, and it will be determined, to a sufficient degree of precision, with four decimals. The average value of the denominator $2m$ is about 80; hence in the numerator it will be sufficiently accurate to put $\rho^2 = 1$, and $2li\zeta = 2mi\zeta$, and neglect the term $- i^2\zeta^2$: and if terms multiplied by i and e^2 are neglected, it is plain that $\zeta = \sin h$. Thus simplified, the equation becomes,

$$\xi \sin M + \eta \cos M = \frac{m^2 - l^2 + 1}{2m} + i \sin h - \frac{1}{2m} \sin^2 h$$

The right-hand member of this equation is a known quantity, and it only remains to discover the expressions of ξ and η in terms of θ to have the equation determining θ.

The known expressions for ξ and η are,

$$\xi = \rho \cos \varphi' \sin \vartheta$$
$$\eta = \rho \cos d \sin \varphi' - \rho \sin d \cos \varphi' \cos \vartheta$$

But if terms of the order of e^4 are neglected,

$$\rho \cos \varphi' = \frac{\cos \varphi}{\rho}$$

$$\rho \sin \varphi' = \frac{(1 - e^2) \sin \varphi}{\rho}$$

Putting $\nu = a' - a$, replacing ϑ by its value $\vartheta' + \nu$, and making $\cos \nu = 1$; since ν is a very small angle, the above expressions become

$$\rho\xi = \cos \varphi \sin \vartheta' + \sin \nu \cos \varphi \cos \vartheta'$$
$$\rho\eta = (1 - e^2) \cos d \sin \varphi - \sin d \cos \varphi \cos \vartheta' + \sin \nu \sin d \cos \varphi \sin \vartheta'$$

In these equations substitute for $\cos \varphi \sin \vartheta'$, $\cos \varphi \cos \vartheta'$, and $\sin \varphi$, their values in terms of θ, which have been given above; then .

$$\rho\xi = \cos h \sin \theta - \sin \nu \sin d' \cos h \cos \theta + \sin \nu \cos d' \sin h$$
$$\begin{aligned}\rho\eta = \sin \nu \sin d \cos h \sin \theta &+ [\cos (d' - d) - e^2 \cos d' \cos d] \cos h \cos \theta \\ &+ [\sin (d' - d) - e^2 \sin d' \cos d] \sin h\end{aligned}$$

But since d' and d are very nearly equal, the last equation may be written

$$\begin{aligned}\rho\eta = \sin \nu \sin d' \cos h \sin \theta &+ (1 - e^2 \cos^2 d') \cos h \cos \theta \\ &+ [\sin (d' - d) - \tfrac{1}{2}e^2 \sin 2 d'] \sin h\end{aligned}$$

Put now

$$\sin \varkappa = \sin \nu \sin d' \qquad K' \sin \varkappa' = \sin (d' - d) - \tfrac{1}{2}e^2 \sin 2d'$$

$$K = 1 - e^2 \cos^2 d' \qquad K' \cos \varkappa' = \sin \nu \cos d'$$

The quantities K, $\varkappa$, K', and $\varkappa'$ will be sensibly constant for the duration of each contact on the earth's surface Then

$$\rho\xi = \cos h \sin \theta - \sin \varkappa \cos h \cos \theta + K' \cos \varkappa' \sin h$$

$$\rho\eta = \sin \varkappa \cos h \sin \theta + K \cos h \cos \theta + K' \sin \varkappa' \sin h$$

Since $\varkappa$ is very small and K nearly unity, there results from these equations,

$$\rho\,[\xi \sin M + \eta \cos M\,] = \sin (M + \varkappa) \cos h \sin \theta + K \cos (M + \varkappa) \cos h \cos \theta$$
$$+ K' \sin (M + \varkappa') \sin h$$

In the next place make

$$L \sin (M + \lambda) = \sin (M + \varkappa)$$

$$L \cos (M + \lambda) = K \cos (M + \varkappa)$$

from which may be derived the sufficiently approximate values,

$$L = 1 - e^2 \cos^2 d' \cos^2 M$$

$$\lambda = \tfrac{1}{2}e^2 \cos^2 d' \sin 2M + \nu \sin d'$$

from which it appears that, since M does not vary much, L and λ are sensibly constant for the duration of each contact on the earth's surface Then putting

$$\gamma = M + \lambda$$

$$\rho\,[\xi \sin M + \eta \cos M\,] = L \cos h \cos (\theta - \gamma) + K' \sin (M + \varkappa') \sin h$$

substituting for $\xi \sin M + \eta \cos M$, its value,

$$\frac{m^2 - l^2 + 1}{2m} + \iota \sin h - \frac{1}{2m} \sin^2 h$$

and making

$$A = \frac{m^2 - l^2 + 1}{2Lm}$$

$$B = \frac{\iota - K' \sin (M + \varkappa')}{L}$$

$$C = -\frac{1}{2Lm}$$

and remembering that ρ and unity may be considered equal when multiplying a small term, the final equation for determining θ is

$$\cos (\theta - \gamma) = \rho \sec h\,[A + B \sin h + C \sin^2 h]$$

This equation possesses the advantage of having its terms separated into factors, one of which depends on the time only, and the other on the altitude only. Thus, in computing the positions of a series of points on a curve of the first class, the quantities A, B, C, and γ, since they are functions of the time only, remain constant. B may be regarded as sensibly constant for the duration of each contact on the earth's surface, and C is nearly so. A, C, and γ are tabulated at intervals of a minute for the duration of each contact on the earth's surface.

The right-hand member of the above equation contains the unknown factor ρ; in a first approximation this will be put equal to unity, and the value of θ thus obtained substituted in the equation,

$$\sin \varphi = \sin d' \sin h + \cos d' \cos h \cos \theta$$

Then a sufficiently accurate value of ρ is given by the equation,

$$\rho = 1 - \tfrac{1}{2} e^2 \sin^2 \varphi$$

However, as four-place logarithms are amply sufficient for all these computations, and the means of estimating the value of φ to within a degree or two are usually not wanting, the repetition of the computation can be avoided. The equation gives two values for θ, corresponding to two points on the earth's surface, which satisfy the conditions of the problem, and ρ must be determined separately for each.

It remains to discover the limits between which the time and the altitude must lie, in order that the solution may be possible. It is evident that when, for a given time, h has its maximum value, the equation determining θ becomes

$$\cos (\theta - \gamma) = \pm 1$$

Thus the condition of contact taking place at maximum altitude is

$$\cos h = \pm \rho \, [A + B \sin h + C \sin^2 h]$$

The ambiguous sign must be so taken that $\cos h$ may be positive. If ρ is put equal to unity or regarded as known, this equation will be of the fourth degree in $\sin h$: but since h must be in the first quadrant it will be found, in general, to have but one root applicable to the problem. It is readily solved by successive approximations; a first value of h may be derived from $\cos h = \pm A$. According as the upper or lower sign has place, the value of θ is γ or $\gamma + 180°$. In this case of maximum altitude, the two solutions of the problem become identical.

Since each curve of contact at the same instant must have two points for $h = 0$, it follows that the time must be so taken that the numerical value of $A\rho$ may not exceed unity. Thus the equations,

$$A\rho = \pm 1$$

give the times of first and last appearance of the contact on the surface of the earth.

In the special case of contact on the horizon, $h = 0$, the equation determining θ takes the simple form,

$$\cos(\theta - \gamma) = A\rho$$

and the equations determining the position of each point reduce to

$$\begin{aligned} \cos\varphi \sin\vartheta' &= \sin\theta \\ \cos\varphi \cos\vartheta' &= -\sin d' \cos\theta \\ \sin\varphi &= \cos d' \cos\theta \\ \omega &= \mu'_1 - \vartheta' \end{aligned}$$

It is worthy of remark that the equation determining θ remains the same if h, instead of being exactly zero, is a small positive or negative angle, for $\sec h$ will be sensibly unity, and, B and C being small, the terms $B \sin h$ and $C \sin^2 h$ may be neglected Hence, in taking into account the effect of refraction on the position of points, where contact takes place in the horizon, θ may still be derived from the equation,

$$\cos(\theta - \gamma) = A\rho$$

but it will be necessary to make $h = -$ (the horizontal refraction) in the equations determining φ and ϑ'

The particular case where $h = 90^\circ$, or contact in the zenith, requires notice Here the equation determining θ reduces to

$$A + B + C = 0$$

This determines the time at which the phenomenon takes place, and the equations for the position of the point reduce to

$$\begin{aligned} \omega &= \mu'_1 \\ \varphi &= d' \end{aligned}$$

PROBLEM II—*To find the point of the earth's surface at which contact takes place at a given point on the sun's limb and at a given altitude*

If the angle of position of the given point measured from the north point of the sun's limb toward the east is denoted by Q, the fundamental eclipse-equations are

$$\begin{aligned} (l - i\zeta) \sin Q &= x - \xi \\ (l - i\zeta) \cos Q &= y - \eta \end{aligned}$$

In these equations $\sin h$ can be substituted for ζ, and x and y can, with sufficient approximation, be represented by the expressions,

$$x = x_0 + \frac{dx}{dt} t$$

$$y = y_0 + \frac{dy}{dt} t$$

it t is counted from an epoch near the middle of the duration of the contact on the earth's surface. Putting now

$$x_0 = m_0 \sin M_0 \qquad \frac{dx}{dt} = n \sin N$$

$$y_0 = m_0 \cos M_0 \qquad \frac{dy}{dt} = n \cos N$$

we have

$$\xi = m_0 \sin M_0 - (l - i \sin h) \sin Q + nt \sin N$$
$$\eta = m_0 \cos M_0 - (l - i \sin h) \cos Q + nt \cos N$$

From these equations are derived the following,

$$\xi \cos N - \eta \sin N = m_0 \sin (M_0 - N) - (l - i \sin h) \sin (Q - N)$$
$$nt = \xi \sin N + \eta \cos N - m_0 \cos (M_0 - N) + (l - i \sin h) \cos (Q - N)$$

The values of ξ and η found in the first problem must be substituted in these equations. The first member of the first of these equations is obtained simply by writing $N + 90°$ for M in the first member of the corresponding equation of the first problem. Hence making

$$L' = 1 - e^2 \cos^2 d' \sin^2 N$$
$$\lambda' = - \tfrac{1}{2} e^2 \cos^2 d' \sin 2N + \nu \sin d'$$
$$\gamma' = N + \lambda' + 90°$$

these quantities are constant for the duration of each contact on the earth's surface, and there is obtained the equation

$$\rho\, [\xi \cos N - \eta \sin N] = L' \cos h \cos (\theta - \gamma') + K' \cos (N + \kappa') \sin h$$

Consequently, if

$$A' = \frac{m_0}{L'} \sin (M_0 - N) - \frac{l}{L'} \sin (Q - N)$$

$$B' = \frac{i}{L'} \sin (M_0 - N) - \frac{K'}{L'} \cos (N + \kappa')$$

where Q has been put equal to M_0 in the term multiplied by i, the equation determining θ in this problem becomes

$$\cos (\theta - \gamma') = \rho \sec h \, [A' + B' \sin h]$$

The equation giving the value of nt is only needed for the purpose of obtaining μ'_1, which it is necessary to have in order to get ω from ϑ'. In this it will be sufficiently accurate to put for ξ and η their approximate values,

$$\xi = \cos h \sin \theta$$
$$\eta = \cos h \sin \theta$$

and neglect the term multiplied by i, then,

$$nt = \cos h \cos (\theta - N) - m_0 \cos (M_0 - N) + l \cos (Q - N)$$

If μ_0 denote the value of μ'_1 at the epoch from which t is counted, μ' the motion of μ'_1 in a unit of time, and

$$A'' = \mu_0 - \frac{m_0\mu'}{n} \cos(M_0 - N) + \frac{l\mu'}{n} \cos(Q - N)$$

the expression for μ'_1 is

$$\mu'_1 = A'' + \frac{\mu'}{n} \cos h \cos(\theta - N)$$

After θ and μ'_1 have been determined from the equations just given, the position of the point on the earth's surface is found by means of the same equations as in the first problem Thus it appears that the solutions of the two problems are quite similar, the only differences being that the term corresponding to $C \sin^2 h$, in the factor of the equation which determines θ, is wanting, and that a separate computation must be made for μ'_1; and the remarks to be made regarding the solution of the equation determining θ, and the limits between which Q and h must be assumed, in order that solution may be possible, are quite similar to those made in the first problem While B' and γ' are constant for the duration of each contact on the earth's surface, A' and A'' involve the variable Q, and may be tabulated with Q as the argument within its limiting values The equation determining θ gives two values for this quantity, corresponding to the two points on the earth's surface, which satisfy the conditions of the problem, and ρ must be determined separately for each

The condition of contact taking place at a given point on the sun's limb, and at the maximum altitude, is

$$\cos h = \pm \rho \, [A' + B' \sin h]$$

and the equations,

$$A'\rho = \pm 1$$

give the limiting values of Q In finding the points on the curves of the second class, which are common to the curve of contact on the horizon, θ is derived from the equation,

$$\cos(\theta - \gamma') = A'\rho$$

but $h = -$ (the horizontal refraction) in the equations which determine φ and ϑ' In computing the value of μ'_1 for each of the two solutions of the problem, it will be noticed that, with sufficient approximation, the second term has the same numerical value but opposite signs in the two solutions, and in the case of maximum altitude for a given value of Q, the equation becomes simply

$$\mu'_1 = A''$$

In this case, also, it will be advantageous to compute four auxiliary quantities from the equations,

$$p \cos \varepsilon = \cos d' \qquad\qquad p' \sin \varepsilon' = \sin d'$$
$$p \sin \varepsilon = \sin d' \cos \theta \qquad\qquad p' \cos \varepsilon' = \cos d' \cos \theta'$$

by means of which the equations determining φ and ϑ' take the form,

$$\cos \varphi \sin \vartheta' = \cos h \sin \theta$$
$$\cos \varphi \cos \vartheta' = p \sin (h - \varepsilon)$$
$$\sin \varphi = p' \cos (h - \varepsilon)$$

As θ is constant in this case, p, p', ε, ε', are so likewise, provided that after the point of maximum altitude has passed the zenith, h be supposed to increase from 90° to 180°, or, in other words, that $180° - h$ be used instead of h.

VALUES OF THE QUANTITIES EMPLOYED.

Denoting the four contacts in their order by the symbols I, II, III, and IV, the values of the various quantities employed in the foregoing discussion are:

	I.	II.	III.	IV.
Epoch from which t is counted	$8^h\ 40^m$	$9^h\ 10^m$	$12^h\ 48^m$	$13^h\ 18^m$
ν	+ 18′	+ 16′	− 6′	− 8′
log K	7.9187	7.9158	7.9359	7.9417
κ'	54° 20′	58° 33′	100° 46′	104° 14′
log L	9.9989	9.9986	9.9977	9.9977
λ	+ 3′	+ 4′	− 3′	− 1′
log E	n 7.1880	7.2228	n 7.3475	n 7.3411
N	284° 50′ 30″.5	284° 48′ 49″.5	284° 36′ 36″.5	284° 34′ 55″.6
L'	9.9977	9.9977	9.9977	9.9977
λ'	− 2′	− 1′	+ 7′	+ 8′
$\frac{m_0}{L} \sin (M_0 - N)$	+ 34.0289	+ 34.0187	+ 34.0188	+ 34.0290
$\log \left[-\frac{l}{L'}\right]$	1.616412	n 1.587591	n 1.587600	1.616423
γ'	14° 49′	14° 48′	14° 44′	14° 43′
log B	7.3788	7.3517	7.3349	n 7.3605
$\mu_0 - \frac{m_0\, u'}{n} \cos (M_0 - N)$	166° 23′	166° 26′	166° 43′	166° 45′
$\log \left[\frac{l\mu'}{n}\right.$ in minutes of arc$\left.\right]$	3.5657	3.5369	3.5368	3.5656
$\log \left[\frac{\mu'}{n}\right.$ in minutes of arc$\left.\right]$	1.9516	1.9516	1.9515	1.9515
log p	9.9978	9.9978	9.9979	9.9979
ε	21° 57′	21° 56′	− 21° 54′	− 21° 54′
log p'	9.9876	9.9876	9.9877	9.9877
ε'	− 156° 40′	− 156° 41′	− 23° 15′	− 23° 15′

The quantities which vary with the time and with Q are given in the following tables

I—*For Exterior Contact at Ingress*

Wash M T	A	log C	γ	ψ'_1	Wash M T	A	log C	γ	ψ'_1
h m			° ′	° ′	h m			° ′	°
8 29	+1 0339	n8 0752	51 29	128 56	8 40	−0 0313	n8 0864	49 25	131 41
30	0 9360	0762	51 13	129 11	41	0 1268	0874	49 13	131 56
31	0 8383	0772	51 7	129 26	42	0 2220	0884	49 1	132 11
32	0 7408	0782	50 56	129 41	43	0 3170	0894	48 50	132 27
33	0 6435	0793	50 44	129 56	44	0 4117	0904	48 38	132 42
34	0 5464	0803	50 33	130 11	45	0 5061	0914	48 26	132 57
35	0 4495	0813	50 22	130 26	46	0 6003	0924	48 14	133 12
36	0 3529	0823	50 11	130 41	47	0 6942	0934	48 2	133 27
37	0 2565	0835	49 59	130 56	48	0 7878	0943	47 50	133 42
38	0 1603	0844	49 48	131 11	49	0 8811	0953	47 38	133 57
39	+0 0644	0854	49 36	131 26	50	0 9742	0963	47 26	134 12
8 40	−0 0313	n8 0864	49 25	131 41	8 51	−1 0670	n8 0973	47 14	134 27

Q	A'	A''	Q	A'	A''	Q	A'	A''
° ′		° ′	° ′		° ′	° ′		° ′
46 50	−1 0360	133 54	48 30	−0 3841	132 26	50 10	+0 2969	130 58
47 0	0 9721	133 45	40	0 3173	132 17	20	0 3666	130 49
10	0 9079	133 36	50	0 2502	132 8	30	0 4366	130 40
20	0 8435	133 27	49 0	0 1828	131 59	40	0 5069	130 31
30	0 7788	133 19	10	0 1152	131 51	50	0 5774	130 23
40	0 7138	133 10	20	−0 0472	131 42	51 0	0 6482	130 14
50	0 6484	133 1	30	+0 0211	131 33	10	0 7193	130 5
48 0	0 5827	132 52	40	0 0897	131 24	20	0 7907	129 56
10	0 5168	132 43	50	0 1585	131 15	30	0 8624	129 47
20	0 4506	132 35	50 0	0 2276	131 7	40	0 9343	129 39
48 30	−0 3841	132 26	50 10	+0 2969	130 58	51 50	+1 0065	129 30

II.—*For Interior Contact at Ingress.*

Wash. M. T.	A	log C	γ	μ'_1	Wash. M. T.	A	log C	γ	μ'_1
h. m.			° ′	° ′	h. m.			° ′	° ′
8 57	+1.0501	n8.1034	46 1	135 57	9 10	−0.0238	n8.1155	43 13	139 13
58	0.9651	.1044	45 48	136 12	11	0.1037	.1164	43 0	139 28
59	0.8807	.1053	45 36	136 27	12	0.1832	.1173	42 47	139 43
9 0	0.7967	.1063	45 23	136 42	13	0.2623	.1181	42 33	139 58
1	0.7130	.1072	45 10	136 57	14	0.3410	.1190	42 20	140 13
2	0.6296	.1081	44 57	137 12	15	0.4193	.1199	42 7	140 28
3	0.5466	.1091	44 45	137 27	16	0.4972	.1208	41 53	140 43
4	0.4640	.1100	44 32	137 42	17	0.5746	.1217	41 39	140 58
5	0.3817	.1109	44 19	137 57	18	0.6515	.1225	41 26	141 13
6	0.2998	.1118	44 6	138 12	19	0.7280	.1234	41 12	141 28
7	0.2183	.1127	43 53	138 28	20	0.8040	.1243	40 58	141 43
8	0.1372	.1137	43 39	138 43	21	0.8795	.1251	40 44	141 58
9	+0.0565	.1146	43 26	138 58	22	0.9546	.1260	40 30	142 13
9 10	−0.0238	8.1155	43 13	139 13	9 23	−1.0292	8.1268	40 17	142 28

Q	A'	A''	Q	A'	A''	Q	A'	A''
° ′		° ′	° ′		° ′	° ′		° ′
39 50	− 1.0401	142 10	42 0	− 0.3965	140 13	44 10	+ 0.2964	138 19
40 0	0.9924	142 1	10	0.3449	140 4	20	0.3517	138 10
10	0.9444	141 52	20	0.2930	139 55	30	0.4073	138 2
20	0.8961	141 43	30	0.2409	139 47	40	0.4632	137 53
30	0.8474	141 34	40	0.1885	139 38	50	0.5194	137 44
40	0.7985	141 25	50	0.1358	139 29	45 0	0.5758	137 35
50	0.7493	141 16	43 0	0.0827	139 20	10	0.6325	137 26
41 0	0.6997	141 7	10	− 0.0294	139 11	20	0.6895	137 18
10	0.6499	140 58	20	+ 0.0242	139 3	30	0.7468	137 9
20	0.5998	140 49	30	0.0781	138 54	40	0.8044	137 1
30	0.5494	140 40	40	0.1322	138 45	50	0.8623	136 52
40	0.4987	140 31	50	0.1866	138 37	46 0	0.9204	136 44
50	0.4477	140 22	44 0	0.2414	138 28	10	0.9788	136 35
42 0	− 0.3965	140 13	44 10	+ 0.2964	138 19	46 20	+ 1.0375	136 27

III—*For Interior Contact at Egress*

Wash M T	A	log C	?	μ'1	Wash M T	A	log C	?	μ1
h m			° ′	° ′	h m			° ′	° ′
12 35	−1 0188	n 8 1276	−10 53	190 41	12 48	−0 0100	n 8 1162	−13 50	193 57
36	0 9440	1268	11 7	190 50	49	+0 0706	1153	14 3	194 12
37	0 8687	1259	11 21	191 11	50	0 1515	1144	14 16	194 27
38	0 7929	1251	11 34	191 26	51	0 2329	1135	14 29	194 42
39	0 7166	1242	11 48	191 41	52	0 3147	1126	14 42	194 57
40	0 6399	1233	12 2	191 56	53	0 3968	1116	14 55	195 12
41	0 5627	1224	12 16	192 11	54	0 4793	1107	15 8	195 27
42	0 4850	1216	12 29	192 26	55	0 5622	1098	15 21	195 42
43	0 4069	1207	12 43	192 41	56	0 6454	1089	15 34	195 57
44	0 3284	1198	12 56	192 56	57	0 7290	1079	15 46	196 12
45	0 2494	1189	13 9	193 11	58	0 8129	1070	15 59	196 27
46	0 1700	1180	13 23	193 27	12 59	0 8972	1060	16 11	196 42
47	0 0902	1171	13 36	193 42	13 0	0 9819	1051	16 24	196 57
12 48	−0 0100	n 8 1162	−13 50	193 57	13 1	+1 0669	n 8 1041	−16 36	197 12

Q	A'	A'	Q	A'	A'	Q	A'	A''
° ′		° ′	° ′		° ′	° ′		° ′
−10 30	− 1 0148	191 4	− 12 40	− 0 3690	193 0	− 14 50	+ 0 3258	194 55
40	0 9669	191 13	50	0 3173	193 9	15 0	0 3813	195 4
50	0 9187	191 22	13 0	0 2653	193 18	10	0 4371	195 13
11 0	0 8702	191 31	10	0 2131	193 27	20	0 4931	195 21
10	0 8215	191 40	20	0 1605	193 36	30	0 5494	195 30
20	0 7724	191 49	30	0 1076	193 44	40	0 6059	195 39
30	0 7230	191 58	40	0 0545	193 53	50	0 6628	195 47
40	0 6733	192 7	50	− 0 0010	194 2	16 0	0 7200	195 56
50	0 6233	192 16	14 0	+ 0 0528	194 11	10	0 7775	196 5
12 0	0 5731	192 25	10	0 1069	194 20	20	0 8353	196 13
10	0 5226	192 34	20	0 1612	194 29	30	0 8933	196 22
20	0 4717	192 43	30	0 2158	194 37	40	0 9515	196 30
30	0 4205	192 51	40	0 2706	194 46	− 16 50	+ 1 0101	196 39
−12 40	− 0 3690	193 0	− 14 50	+ 0 3258	194 55			

IV.—*For Exterior Contact at Egress.*

Wash. M. T.	A	log C	ζ	p'_1	Wash. M. T.	A	log C	ζ	p'_1
h. m.			° ′	° ′	h. m.			° ′	° ′
13 7	−1.0536	n 8.0983	−17 48	198 43	13 18	−0.0144	n 8.0874	−19 59	201 29
8	0.9605	.0974	18 1	198 58	19	+0.0817	.0864	20 10	201 44
9	0.8671	.0964	18 13	199 13	20	0.1780	.0854	20 22	201 59
10	0.7735	.0954	18 25	199 28	21	0.2745	.0844	20 33	202 14
11	0.6795	.0944	18 37	199 44	22	0.3713	.0834	20 45	202 29
12	0.5853	.0934	18 49	199 59	23	0.4683	.0823	20 56	202 44
13	0.4908	.0924	19 0	200 14	24	0.5655	.0813	21 8	202 59
14	0.3960	.0914	19 12	200 29	25	0.6629	.0803	21 19	203 14
15	0.3010	.0904	19 24	200 44	26	0.7605	.0793	21 30	203 29
16	0.2057	.0894	19 36	200 59	27	0.8583	.0783	21 41	203 44
17	0.1102	.0884	19 47	201 14	28	0.9563	.0772	21 53	203 59
13 18	−0.0144	n 8.0874	−19 59	201 29	13 29	+1.0545	n 8.0762	−22 1	204 14

Q	A'	A''	Q	A'	A''	Q	A'	A''
° ′		° ′	° ′		° ′	° ′		° ′
−17 30	− 1.0021	199 18	− 19 10	− 0.3487	200 48	− 20 50	+ 0.3341	202 16
40	0.9380	199 27	20	0.2817	200 57	21 0	0.4039	202 25
50	0.8737	199 36	30	0.2144	201 6	10	0.4739	202 34
18 0	0.8091	199 45	40	0.1468	201 15	20	0.5443	202 42
10	0.7443	199 54	50	0.0790	201 24	30	0.6150	202 51
20	0.6791	200 3	20 0	− 0.0109	201 33	40	0.6859	203 0
30	0.6136	200 12	10	+ 0.0575	201 41	50	0.7572	203 8
40	0.5478	200 21	20	0.1262	201 50	22 0	0.8288	203 17
50	0.4817	200 30	30	0.1952	201 59	10	0.9006	203 26
19 0	0.4153	200 39	40	0.2645	202 8	20	0.9727	203 34
−19 10	− 0.3487	200 48	− 20 50	+ 0.3341	202 16	− 22 30	+ 1.0151	203 43

BEGINNING, ETC, OF EACH CONTACT

From the foregoing data are readily derived the times, and position of the places, at which the following phenomena occur

		Wash M T		Longitude		Latitude		
		h	m		′		°	′
Contact I	begins on the earth	8	29 335	55	27	+	35	24
	occurs in the zenith	8	39 530	131	34	−	22	38
	ends on the earth	8	50 292	244	25	−	38	24
Contact II	begins on the earth	8	57 572	65	53	+	40	15
	occurs in the zenith	9	9 520	139	6	−	22	37
	ends on the earth	9	22 630	257	24	−	44	22
Contact III	begins on the earth	12	35 216	36	40	−	64	33
	occurs in the zenith	12	48 314	194	2	−	22	34
	ends on the earth	13	0 244	235	15	+	62	48
Contact IV	begins on the earth	13	7 548	38	15	−	61	0
	occurs in the zenith	13	18 300	201	33	−	22	34
	ends on the earth	13	28 471	251	17	+	59	20

APPROXIMATION OF THE CURVES TO CIRCLES

The curves to be drawn on the charts approximate so closely to circles of the sphere that it has been deemed sufficient to compute the positions of three points on each curve, namely, the two at which contact occurs on the horizon, and the one for which the altitude is a maximum, and then regard the curve as a circle of the sphere passing through these points, and, as the stereographic projection has been chosen for the delineation of the charts, the projected curves will also be circles

But it will be of interest to determine beforehand how great an error can be produced by this assumption And first, in the case of the time-curves, let σ be the radius of the circle of the sphere passing through the three points, and adopt the subscripts (0), (1), (2), (3), for the quantities which refer respectively to the pole of this circle, the points of contact on the horizon, and the point of maximum altitude Then σ and the position of the pole of this circle are determined by the equations,

$$\sin\varphi_1 \sin\varphi_0 + \cos\varphi_1 \cos\varphi_0 \cos(\vartheta'_1 - \vartheta'_0) = \cos\sigma$$
$$\sin\varphi_2 \sin\varphi_0 + \cos\varphi_2 \cos\varphi_0 \cos(\vartheta'_2 - \vartheta'_0) = \cos\sigma$$
$$\sin\varphi_3 \sin\varphi_0 + \cos\varphi_3 \cos\varphi_0 \cos(\vartheta'_3 - \vartheta'_0) = \cos\sigma$$

or, if for the moment we write, in general,

$$x = \cos\varphi \sin\vartheta'$$
$$y = \cos\varphi \cos\vartheta'$$
$$z = \sin\varphi$$

by the equations,

$$x_1 x_0 + y_1 y_0 + z_1 z_0 = \cos\sigma$$
$$x_2 x_0 + y_2 y_0 + z_2 z_0 = \cos\sigma$$
$$x_3 x_0 + y_3 y_0 + z_3 z_0 = \cos\sigma$$

It will be sufficient to assume here that the circle which passes through the point of maximum altitude and the two points for which $h = -$ (the horizontal refraction) will also pass through the two points for which $h = 0$. Consequently, we shall suppose that $h_1 = 0$ and $h_2 = 0$. But, from the foregoing investigation,

$$x_1 = \sin \theta_1 \qquad x_2 = \sin \theta_2$$
$$y_1 = -\sin d' \cos \theta_1 \qquad y_2 = -\sin d' \cos \theta_2$$
$$z_1 = \cos d' \cos \theta_1 \qquad z_2 = \cos d' \cos \theta_2$$

$$x_3 = \cos h_3 \sin \theta_3$$
$$y_3 = \cos d' \sin h_3 - \sin d' \cos h_3 \cos \theta_3$$
$$z_3 = \sin d' \sin h_3 + \cos d' \cos h_3 \cos \theta_3$$

and if two unknowns, v and τ, are taken so that

$$x_0 = \sin v$$
$$y_0 = -\sin (d' - \tau) \cos v$$
$$z_0 = \cos (d' - \tau) \cos v$$

the equations determining σ, v, and τ are

$$\sin \theta_1 \sin v + \cos \theta_1 \cos v \cos \tau = \cos \sigma$$
$$\sin \theta_2 \sin v + \cos \theta_2 \cos v \cos \tau = \cos \sigma$$
$$\cos h_3 \sin \theta_3 \sin v + \cos h_3 \cos \theta_3 \cos v \cos \tau + \sin h_3 \cos v \sin \tau = \cos \sigma$$

from which are derived

$$\sec \sigma \sin v = \frac{\sin \frac{1}{2} (\theta_2 + \theta_1)}{\cos \frac{1}{2} (\theta_2 - \theta_1)}$$

$$\sec \sigma \cos v \cos \tau = \frac{\cos \frac{1}{2} (\theta_2 + \theta_1)}{\cos \frac{1}{2} (\theta_2 - \theta_1)}$$

$$\sec \sigma \cos v \sin \tau = \frac{1}{\sin h_3} \left[1 - \cos h_3 \frac{\cos \frac{1}{2} (\theta_2 + \theta_1 - 2\theta_3)}{\cos \frac{1}{2} (\theta_2 - \theta_1)} \right]$$

But since θ_1 and θ_2 are given by the equations,

$$\cos (\theta_1 - \gamma) = A\rho_1 = A \left(1 - \frac{e^2}{2} \cos^2 d' \cos^2 \theta_1\right)$$

$$\cos (\theta_2 - \gamma) = A\rho_2 = A \left(1 - \frac{e^2}{2} \cos^2 d' \cos^2 \theta_2\right)$$

where $\theta_2 - \gamma$ is nearly $360° - \theta_1 + \gamma$, we shall have

$$\cos \left[\frac{\theta_2 + \theta_1}{2} - \gamma \right] \cos \frac{\theta_2 - \theta_1}{2} = \pm A \left[1 - \frac{e^2}{2} \cos^2 d' \, (A^2 \cos 2\gamma + \sin^2 \gamma) \right]$$

$$\sin \left[\frac{\theta_2 + \theta_1}{2} - \gamma \right] = \mp \tfrac{1}{2} A^2 e^2 \cos^2 d' \sin 2\gamma$$

As for the ambiguous signs, they are determined by the following conditions Let it be agreed that the position of the pole, for which σ is less than 90°, is to be found And as the equations ought not to be changed when the subscripts (1) and (2) are interchanged, let

$$\beta = \frac{\theta_2 \sim \theta_1}{2}$$

be so taken that β is in the first quadrant, and let

$$\gamma_0 = \frac{\theta_2 + \theta_1}{2}$$

be taken in that quadrant which makes $\frac{\theta_2 + \theta_1}{2} - \theta_3$ a small positive, or negative, angle, then

$$\gamma_0 = \gamma - \tfrac{1}{4} A^2 e^2 \cos^2 d' \sin 2\gamma$$

when A is positive, and this expression augmented by 180°, when A is negative, and

$$\cos \beta = \pm A \left[1 - \frac{e^2}{2} \cos^2 d' \left(A^2 \cos^2 \gamma + \sin^2 \gamma \right) \right]$$

the ambiguous sign to be so taken that $\cos \beta$ may be positive The quantity $\cos\left(\frac{\theta_2 + \theta_1}{2} - \theta_3\right)$ differs from unity by a quantity of the order of e^4, which may be neglected Moreover, h_3 and β are nearly equal Thus, the equations determining σ, v, and τ take the simpler forms,

$$\sec \sigma \sin v = \frac{\sin \gamma_0}{\cos \beta}$$

$$\sec \sigma \cos v \cos \tau = \frac{\cos \gamma_0}{\cos \beta}$$

$$\sec \sigma \cos v \sin \tau = \frac{h_3 - \beta}{\cos \beta}$$

Since $h_3 - \beta$ is small, its square may be neglected, and the equations give

$$\sigma = \beta$$
$$v = \gamma_0$$
$$\tau = \frac{h_3 - \beta}{\cos \gamma_0}$$

whence τ is a small positive or negative angle The position of the pole of the circle is then given by the equations,

$$\cos \varphi_0 \cos \vartheta'_0 = -\cos \gamma_0 \sin (d' - \tau)$$
$$\cos \varphi_0 \sin \vartheta_0 = \sin \gamma_0$$
$$\sin \varphi_0 = \cos \gamma_0 \cos (d' - \tau)$$
$$\omega_0 = \mu'_1 - \vartheta'_0$$

If the distance of any point on the time-curve from this pole be denoted by σ', then $\sigma' - \sigma$ may be taken as a sufficiently exact measure of the error committed by our method of drawing the curve

But

$$\cos\sigma' = xx_0 + yy_0 + zz_0$$
$$x = \cos h \sin\theta$$
$$y = \cos d' \sin h - \sin d' \cos h \cos\theta$$
$$z = \sin d' \sin h + \cos d' \cos h \cos\theta$$

whence

$$\cos\sigma' = \cos h \sin\theta \sin\gamma_0 + \cos h \cos\theta \cos\gamma_0 \cos\tau + \sin h \cos\gamma_0 \sin\tau$$

or, as $\cos\tau$ may be put equal to unity,

$$\cos\sigma' = (h_3 - \beta)\sin h + \cos h \cos(\theta - \gamma_0)$$

The quantity $\sigma' - \sigma$ is composed of two parts independent of each other, the first depending on the curvature of the cone enveloping the sun and Venus, and proportional to the quantity we have denoted by C, the second due to the non-sphericity of the earth and proportional to e^2 These parts can then be determined separately

First, from the equations,

$$\cos h_3 = \pm(A + B\sin h_3 + C\sin^2 h_3)$$
$$\cos\beta = \pm A$$

is obtained, with sufficient exactness,

$$h_3 - \beta = \mp(B + C\sin\beta)$$

But

$$\cos h \cos(\theta - \gamma_0) = \pm(A + B\sin h + C\sin^2 h)$$
$$\cos\sigma = \pm A$$

thus

$$\cos\sigma' - \cos\sigma = \pm C\sin h(\sin h - \sin\beta)$$

Secondly, from the equations,

$$\cos h_3 = \pm A\rho_3 = \pm A\left[\mathrm{I} - \frac{e^2}{2}(\sin d' \sin\beta \pm \cos d' \cos\beta\cos\gamma)^2\right]$$

$$\cos\beta = \pm A\left[\mathrm{I} - \frac{\mathrm{I}}{2}e^2\cos^2 d'(A^2\cos 2\gamma + \sin^2\gamma)\right]$$

we find that the part of $h_3 - \beta$ proportional to e^2 is

$$h_3 - \beta = \frac{\mathrm{I}}{2}e^2\cos\beta\left[\sin^2 d'\sin\beta \pm \sin 2d'\cos\beta\cos\gamma - \cos^2 d'\sin\beta\sin^2\gamma\right]$$

Also

$$\cos h \cos(\theta - \gamma_0) = \pm A\,\rho \mp \tfrac{1}{2}e^2 \cos^2 d' \sin 2\gamma \cos^2\beta \cos h \sin(\theta-\gamma)$$

$$= \pm A\left[\,1 - \frac{e^2}{2}(\sin d' \sin h + \cos d' \cos h \cos\theta)^2\right]$$

$$\mp \frac{e^2}{2}\cos^2 d' \sin 2\gamma \cos^2\beta \sqrt{(\sin^2\beta - \sin^2 h)}$$

$$\cos h \cos\theta = A\cos\gamma - \sqrt{(\sin^2\beta - \sin^2 h)}\sin\gamma$$

where the sign of $\sin(\theta-\gamma)$ must be attributed to the radical $\sqrt{(\sin^2\beta - \sin^2 h)}$

After some reductions it will be found that

$$\cos\sigma' - \cos\sigma = \frac{e^2}{2}(\sin^2 d' - \cos^2 d' \sin^2\gamma)\cos\beta \sin h(\sin\beta - \sin h)$$

$$+\frac{e^2}{2}\sin 2d' \sin\gamma \cos\beta \sin h \sqrt{(\sin^2\beta - \sin^2 h)}$$

Uniting to this the term proportional to C, we have the complete value

$$\cos\sigma' - \cos\sigma = \left[\frac{e^2}{2}(\sin^2 d' - \cos^2 d' \sin^2\gamma)\cos\beta \mp C\right](\sin\beta - \sin h)\sin h$$

$$+\frac{e^2}{2}\sin 2d' \sin\gamma \cos\beta \sin h \sqrt{(\sin^2\beta - \sin^2 h)}$$

It will be seen that this expression vanishes when $h = 0$ and when $h = \beta$, as it should Differentiating this expression with respect to the variable $\sin h$, in order to obtain its maximum value, we arrive at an equation of the fourth degree in $\sin h$ Hence we are obliged to content ourselves with a superior limit to the maximum value, which, however, for practical purposes, may be regarded as identical with it The first term of the expression has its maximum value when $\sin h = \frac{1}{2}\sin\beta$, and the second when $\sin h = \frac{1}{\sqrt{2}}\sin\beta$ Substituting these values in their respective terms, we obtain

$$\sigma' - \sigma = \frac{e^2}{16}(\cos^2 d' \sin^2\gamma \pm 2\sin 2d' \sin\gamma - \sin^2 d')\sin 2\beta \pm \frac{C}{4}\sin\beta$$

where the ambiguous signs, in both cases, must be so taken that the largest numerical value of the expression will be obtained Replacing e^2 and C by their values, and taking for the factor which involves d' and γ the greatest value it can have, it results that $\sigma' - \sigma$ cannot exceed

$$11' \sin\beta + 2' \sin 2\beta$$

and the maximum value of this with regard to the variable β is less than $12'$ Having

regard to the scale on which the charts have been constructed, this quantity may be considered as within the unavoidable errors produced by imperfection of drawing.

It is worthy of remark that, in our method of drawing the curves, the error is only a fourth part of that which results from neglecting altogether the curvature of the cones enveloping the sun and the planet, as has generally been done in treatises on practical astronomy.

The investigation of the error in the case of the second class of curves differs somewhat from that of the first class, on account of μ'_1 not being constant for all points on the curve. The equations determining σ and the position of the pole are

$$\sin \varphi_1 \sin \varphi_0 + \cos \varphi_1 \cos \varphi_0 \cos (\omega_1 - \omega_0) = \cos \sigma$$
$$\sin \varphi_2 \sin \varphi_0 + \cos \varphi_2 \cos \varphi_0 \cos (\omega_2 - \omega_0) = \cos \sigma$$
$$\sin \varphi_3 \sin \varphi_0 + \cos \varphi_3 \cos \varphi_0 \cos (\omega_3 - \omega_0) = \cos \sigma$$

where

$$\omega_1 = A'' - \frac{\mu'}{n} \sin (\theta_1 - \gamma') - \vartheta'_1$$

$$\omega_2 = A'' + \frac{\mu'}{n} \sin (\theta_1 - \gamma') - \vartheta'_2$$

$$\omega_3 = A'' - \vartheta'_3$$

If we put

$$g = \frac{\mu'}{n} \sin (\theta_1 - \gamma') = \pm \frac{\mu'}{n} \sqrt{(1 - A'^2)}$$

$$\omega_0 = A'' - \vartheta'_0$$

g is a small angle, whose square may be neglected, and the equations, using the notation given in the case of the first class of curves, take the shape,

$$(x_1 + g y_1)\, x_0 + (y_1 - g x_1)\, y_0 + z_1 z_0 = \cos \sigma$$
$$(x_2 - g y_2)\, x_0 + (y_2 + g x_2)\, y_0 + z_2 z_0 = \cos \sigma$$
$$x_3 x_0 + y_3 y_0 + z_3 z_0 = \cos \sigma$$

Put

$$\theta'_1 = \theta_1 - g \sin d'$$
$$\theta'_2 = \theta_2 + g \sin d'$$

and, as τ is here also a small angle, making $\cos \tau = 1$, the equations, using the same notation as before, become

$$\sin \theta'_1 \sin v + \cos \theta'_1 \cos v = \cos \sigma$$
$$\sin \theta'_2 \sin v + \cos \theta'_2 \cos v = \cos \sigma$$
$$\cos h_3 \sin \theta_3 \sin v + \cos h_3 \cos \theta_3 \cos v + \sin h_3 \cos v \sin \tau = \cos \sigma$$

These are entirely similar to the analogous equations for the first class of curves. Hence the operations here being identical with those of the former case, it will be necessary to note only the final results. If

$$\gamma_0 = \gamma' - \tfrac{1}{2}A'c^2 \cos^2 d' \sin 2\gamma'$$

when A' is positive, and this expression augmented by 180° when A' is negative, and

$$\cos\beta = \pm A' \left[1 - \frac{c^2}{2}\cos^2 d' \left(A'^2 \cos 2\gamma' + \sin^2\gamma'\right) \right] \pm \frac{\mu'}{n}\left(1 - A'^2\right)\sin d'$$

the upper or lower signs being taken so as to render $\cos\beta$ positive, then

$$\sigma = \beta$$
$$\tau = \frac{h_3 - \beta}{\cos\gamma_0}$$

and the position of the pole of the circle is given by the equations,

$$\cos\varphi_0 \sin\vartheta'_0 = \sin\gamma_0$$
$$\cos\varphi_0 \cos\vartheta'_0 = -\cos\gamma_0 \sin(d' - \tau)$$
$$\sin\varphi_0 = \cos\gamma_0 \cos(d' - \tau)$$
$$\omega_0 = A'' - \vartheta'_0$$

To determine the error of representing this class of curves by circles of the sphere,

$$\cos\sigma' = \sin\varphi\sin\varphi_0 + \cos\varphi\cos\varphi_0\cos(\omega - \omega_0)$$
$$\omega = A'' - \frac{\mu'}{n}\cos h \sin(\theta - \gamma') - \vartheta'$$

whence

$$\cos\sigma' = xx_0 + yy_0 + zz_0 + \frac{\mu'}{n}\cos h\sin(\theta - \gamma')(yx_0 - xy_0)$$

$$= (h_3 - \beta)\sin h + \cos h\cos(\theta - \gamma_0)$$
$$+ \frac{\mu'}{n}\sqrt{(\sin^2\beta - \sin^2 h)}\left[\cos d' \sin\gamma_0 \sin h + \sin d' \cos h \sin(\theta - \gamma_0)\right]$$

$$= (h_3 - \beta)\sin h + \cos h\cos(\theta - \gamma_0) \pm \frac{\mu'}{n}\sin d' \left(\sin^2\beta - \sin^2 h\right)$$

$$\pm \frac{\mu'}{n}\cos d' \sin\gamma' \sin h\sqrt{(\sin^2\beta - \sin^2 h)}$$

where the upper or lower sign is taken according as A' is positive or negative, and the sign of $\sin(\theta - \gamma')$ is assigned to the radical

$$\sqrt{(\sin^2 \beta - \sin^2 h)}$$

The part of $\cos\sigma' - \cos\sigma$ which involves the factor $\frac{\mu'}{n}$ will be found to be

$$\pm \frac{\mu'}{n} \sin d' \sin h \, (\sin\beta - \sin h)$$

$$\pm \frac{\mu'}{n} \cos d' \sin\gamma' \sin h \sqrt{(\sin^2\beta - \sin^2 h)}$$

The part proportional to e^2 is obtained from the analogous expression in the case of the first class of curves, simply by changing γ into γ', and thus is

$$\frac{e^2}{2} (\sin^2 d' - \cos^2 d' \sin^2 \gamma') \cos\beta \sin h \, (\sin\beta - \sin h)$$

$$+ \frac{e^2}{2} \sin 2d' \sin\gamma' \cos\beta \sin h \sqrt{(\sin^2\beta - \sin^2 h)}$$

Combining these two parts, we have

$$\cos\sigma' - \cos\sigma = \left[\frac{e^2}{2} (\sin^2 d' - \cos^2 d' \sin^2\gamma') \cos\beta \pm \frac{\mu'}{n} \sin d'\right] (\sin\beta - \sin h) \sin h$$

$$+ \left[\frac{e^2}{2} \sin 2d' \sin\gamma' \cos\beta \pm \frac{\mu'}{n} \cos d' \sin\gamma'\right] \sin h \sqrt{(\sin^2\beta - \sin^2 h)}$$

Deriving a superior limit to the maximum value of $\sigma' - \sigma$ by the same method as in the former case, it is found to be, with regard to the variable h,

$$\sigma' - \sigma = -\frac{e^2}{16} (\sin^2 d' \pm 2 \sin 2d' \sin\gamma' - \cos^2 d' \sin^2\gamma') \sin 2\beta$$

$$\pm \frac{\mu'}{4n} (\sin d' \pm 2 \cos d' \sin\gamma') \sin\beta$$

where the ambiguous signs must be taken so as to make the numerical value of the expression the largest. On substituting the numerical values of d' and γ', it will be seen that the term proportional to e^2 has no appreciable effect in augmenting the maximum value of $\sigma' - \sigma$, which is found to be $18'$.

POSITIONS OF POINTS OF THE CURVES.

The positions of the points needed for drawing the curves are given below: for the two points on the horizon $h = -35'$; and for the point of maximum altitude, the value of this quantity is given in the last column.

I.—*Exterior Contact at Ingress*

FIRST CLASS OF CURVES

Wash M T	Contact on the horizon				Contact at maximum altitude		
	Long	Lat	Long	Lat	Long	Lat	Max alt
h m	° ′	° ′	° ′	° ′	° ′	° ′	° ′
8 30	71 42	+ 53 8	45 47	+ 16 52	76 32	+ 23 37	21 2
31	88 46	61 56	41 9	+ 5 37	87 4	15 41	33 32
32	107 51	66 27	37 56	− 2 40	94 22	9 32	42 45
33	128 12	67 56	35 14	9 37	100 23	+ 4 14	50 34
34	146 54	67 3	32 45	15 48	105 42	− 0 34	57 32
36	173 35	61 7	27 48	26 48	115 18	9 14	70 5
38	189 18	52 40	22 19	36 37	124 27	17 3	81 33
40	199 29	43 19	15 25	45 40	133 50	24 18	87 25
42	207 4	33 18	5 45	54 4	144 9	31 8	76 24
44	213 24	22 36	350 40	61 25	156 15	37 22	64 58
46	219 20	10 42	325 43	66 17	171 29	42 50	52 26
47	222 25	+ 3 59	308 46	66 43	181 0	45 1	45 27
48	225 50	− 3 36	290 27	64 58	192 25	46 32	37 31
49	230 1	12 46	272 38	60 14	206 51	46 52	27 49
8 50	237 1	− 26 41	254 41	− 49 38	228 10	− 43 57	12 17

SECOND CLASS OF CURVES

Angle of position of point of contact	Contact on the horizon				Contact at maximum altitude		
	Long	Lat	Long	Lat	Long	Lat	Max alt
° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
47 0	259 46	− 53 33	311 52	− 66 46	252 16	− 73 36	14 10
47 20	244 42	36 21	351 59	60 57	182 55	73 32	32 37
47 40	237 41	27 52	7 26	52 52	158 46	65 0	44 28
48 0	232 58	18 55	16 1	45 1	148 31	56 25	54 19
48 20	229 7	10 47	21 46	37 29	142 35	48 19	63 8
48 40	225 39	− 3 10	26 5	30 11	138 27	40 31	71 23
49 0	222 20	+ 4 11	29 37	23 2	135 13	32 55	79 20
49 20	219 1	11 24	32 44	15 54	132 30	25 23	87 10
49 40	215 29	18 37	35 37	8 41	130 3	17 48	85 0
50 0	211 36	25 54	38 29	− 1 19	127 46	10 1	76 59
50 20	207 2	33 24	41 28	+ 6 24	125 28	− 1 55	68 38
50 40	201 16	41 14	44 49	14 38	123 3	+ 6 47	59 41
51 0	193 11	49 34	48 57	23 13	120 19	16 26	49 45
51 20	179 34	58 32	54 46	34 20	116 45	27 51	37 56
51 40	147 8	+ 67 4	66 17	+ 48 37	110 22	+ 43 47	21 16

II.—*Interior Contact at Ingress.*

FIRST CLASS OF CURVES.

Wash. M. T.	Contact on the horizon.				Contact at maximum altitude.		
	Long.	Lat.	Long.	Lat.	Long.	Lat.	Max. alt.
h. m.	° ′	° ′	° ′	° ′	° ′	° ′	° ′
8 58	78 40	+ 53 6	57 25	+ 26 25	82 32	+ 31 1	15 42
59	96 45	62 17	52 8	14 50	93 49	22 24	28 46
9 0	115 41	66 34	48 57	6 54	100 51	16 5	37 46
1	135 13	67 57	46 28	+ 0 22	106 20	10 45	45 8
2	152 56	67 12	44 20	− 5 21	111 3	+ 5 57	51 40
4	178 21	61 57	40 27	15 23	119 11	− 2 36	63 7
6	193 36	54 37	36 40	24 16	126 31	10 17	73 21
8	203 36	46 37	32 36	32 24	133 37	17 26	82 56
10	210 52	38 17	27 48	40 5	140 52	24 13	87 48
12	216 41	29 45	21 43	47 25	148 41	30 42	78 38
14	221 45	20 53	13 17	54 24	157 33	36 56	69 18
16	226 32	11 26	0 41	60 45	168 15	42 51	59 30
18	231 24	+ 1 4	340 39	65 33	182 3	48 11	48 47
19	234 8	− 4 52	326 47	66 42	190 51	50 27	42 47
20	237 5	11 10	310 30	66 23	201 28	52 11	36 5
21	240 45	18 42	293 3	63 53	214 42	52 57	28 10
9 22	246 12	− 28 48	275 6	− 57 43	232 36	− 51 42	17 20

SECOND CLASS OF CURVES.

Angle of position of point of contact.	Contact on the horizon.				Contact at maximum altitude.		
	Long.	Lat.	Long.	Lat.	Long.	Lat.	Max. alt.
° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
40 0	275 5	− 57 43	305 33	− 65 56	274 58	− 69 48	8 15
40 20	256 41	43 33	349 9	64 4	212 8	75 58	26 37
40 40	249 44	34 27	6 45	58 10	179 26	70 41	37 7
41 0	245 5	26 50	16 36	51 59	165 11	64 1	45 38
41 20	241 26	20 1	23 3	46 1	157 24	57 30	53 7
41 40	238 17	13 41	27 45	40 9	152 16	51 10	60 2
42 0	235 26	7 40	31 25	34 29	148 30	45 5	66 33
42 20	232 45	− 1 50	34 27	28 53	145 29	39 5	72 52
42 40	230 7	+ 3 51	37 7	23 19	142 58	33 12	79 1
43 0	227 28	9 30	39 31	17 45	140 44	27 19	85 8
43 20	224 43	15 8	41 47	12 8	138 42	21 24	88 48
43 40	221 48	20 50	43 57	6 25	136 47	15 23	82 32
44 0	218 35	26 36	46 9	− 0 33	134 55	9 13	76 10
44 20	214 55	32 34	48 27	+ 5 34	133 2	− 2 46	69 32
44 40	210 31	38 45	51 0	12 1	131 6	+ 4 2	62 32
45 0	204 57	45 13	53 55	18 57	128 59	11 22	54 59
45 20	197 14	52 6	57 33	26 37	126 33	19 32	46 34
45 40	184 50	59 24	62 36	35 29	123 26	29 8	36 38
46 0	159 13	+ 66 27	71 37	+ 46 59	118 23	+ 41 49	23 22

III.—Interior Contact at Egress

FIRST CLASS OF CURVES

Wash. M. T.	Contact on the horizon				Contact at maximum altitude		
	Long	Lat	Long	Lat	Long	Lat	Max. alt.
h m	° ′	° ′	° ′	° ′	° ′	° ′	° ′
12 36	67 18	− 52 11	349 56	− 65 24	86 11	− 79 10	19 26
37	76 50	43 42	330 38	60 36	137 51	77 10	29 31
38	82 28	36 50	320 13	55 34	159 44	72 18	37 11
39	86 37	30 45	313 26	50 46	169 51	66 37	43 49
40	89 57	25 11	308 35	46 10	175 36	61 19	49 39
42	95 23	14 59	301 48	37 24	182 34	51 18	60 16
44	100 0	− 5 30	297 4	29 7	187 2	41 55	70 2
46	104 18	+ 3 33	293 21	21 4	190 32	32 53	79 22
48	108 34	12 28	290 8	13 3	193 34	23 59	88 33
50	113 7	21 22	287 8	− 4 54	196 27	14 57	82 9
52	118 21	30 28	284 5	+ 3 36	199 21	− 5 35	72 30
54	124 57	39 57	280 44	12 41	202 29	+ 4 26	62 9
56	134 33	50 2	276 33	22 51	206 9	15 38	50 31
57	141 44	55 19	273 50	28 38	208 24	22 0	43 52
58	152 26	60 44	270 13	35 12	211 12	29 18	36 13
59	170 41	65 49	264 43	43 10	215 9	38 10	26 50
13 0	209 26	+ 67 32	251 50	+ 55 3	223 42	+ 52 6	11 45

SECOND CLASS OF CURVES

Angle of position of point of contact	Contact on the horizon				Contact at maximum altitude		
	Long	Lat	Long	Lat	Long	Lat	Max. alt.
° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
− 10 40	316 1	− 52 46	12 28	− 66 50	306 6	− 74 19	15 20
11 0	304 19	40 59	45 8	62 36	250 1	74 50	29 43
11 20	298 50	32 24	60 29	56 30	224 56	68 57	39 30
11 40	295 7	25 2	69 45	50 22	213 42	62 15	47 41
12 0	292 15	18 23	76 14	44 24	207 27	55 45	55 0
12 20	289 48	12 8	81 10	38 36	203 23	49 30	61 47
12 40	287 35	6 10	85 13	32 56	200 27	43 25	68 15
13 0	285 31	− 0 23	88 44	27 20	198 13	37 25	74 31
13 20	283 29	+ 5 18	91 53	21 47	196 23	31 35	80 30
13 40	281 24	10 57	94 48	16 13	194 48	25 12	86 45
14 0	279 13	16 36	97 37	10 34	193 24	19 45	87 6
14 20	276 50	22 18	100 22	− 4 49	192 6	13 42	80 51
14 40	274 5	28 8	103 10	+ 1 7	190 49	7 28	74 26
15 0	270 53	34 8	106 6	7 19	189 32	− 0 55	67 42
15 20	266 19	40 24	109 17	13 52	188 8	+ 6 0	60 35
15 40	261 25	46 58	112 56	20 58	186 34	13 31	52 50
16 0	253 24	53 59	117 26	28 54	184 34	21 59	44 6
16 20	239 27	61 22	123 41	38 15	181 43	32 9	33 34
− 16 40	206 43	+ 67 44	135 51	+ 51 6	175 56	+ 46 36	18 22

IV.—*Exterior Contact at Egress*

FIRST CLASS OF CURVES

Wash. M. T.		Contact on the horizon								Contact at maximum altitude					
		Long.		Lat.		Long.		Lat.		Long.		Lat.		Max. alt.	
h	m	°	′	°	′	°	′	°	′	°	′	°	′	°	′
13	8	79	5	− 49	12	22	19	− 66	49	92	22	− 71	58	16	27
	9	89	39	37	40	350	56	64	5	136	44	72	10	29	44
	10	95	17	29	12	335	3	59	2	159	56	67	18	38	58
	11	99	23	21	58	325	34	53	46	172	21	61	35	46	42
	12	102	43	15	28	319	2	48	27	180	2	55	52	53	35
	14	108	21	− 3	35	310	30	38	19	189	34	44	54	65	56
	16	113	25	+ 7	22	304	48	28	34	195	52	34	25	77	20
	18	118	31	17	53	300	20	18	54	200	52	24	7	88	21
	20	124	11	28	19	296	27	− 9	7	205	21	13	42	80	34
	22	131	13	38	54	292	37	+ 1	12	209	48	− 2	47	68	59
	24	141	26	49	54	288	21	12	30	214	41	+ 9	9	56	16
	25	149	7	55	33	285	46	18	49	217	33	15	48	49	9
	26	160	34	61	11	282	33	25	55	220	59	23	17	41	5
	27	179	43	66	10	278	5	34	18	225	35	32	8	31	27
13	28	215	33	+ 67	37	269	28	+ 46	2	233	48	+ 41	25	17	40

SECOND CLASS OF CURVES

Angle of position of point of contact		Contact on the horizon								Contact at maximum altitude					
		Long.		Lat.		Long.		Lat.		Long.		Lat.		Max. alt.	
°	′	°	′	°	′	°	′	°	′	°	′	°	′	°	′
− 17	40	319	8	− 48	33	33	31	− 66	10	294	5[illegible]	− 76	11	20	36
18	0	308	37	35	23	63	48	58	49	239	29	71	16	36	16
18	20	303	17	25	27	77	16	50	43	222	6	62	39	47	14
18	40	299	26	16	44	85	26	42	56	214	8	54	12	56	43
19	0	296	21	8	50	91	17	35	27	209	26	46	8	65	23
19	20	293	33	− 1	18	95	55	28	12	206	13	38	24	73	32
19	40	290	50	+ 6	2	99	54	21	3	203	47	30	50	81	26
20	0	288	5	13	14	103	30	13	54	201	46	23	17	89	15
20	20	285	5	20	27	106	56	− 6	40	199	58	15	40	82	53
20	40	281	39	27	48	110	24	+ 0	48	198	17	− 7	49	74	48
21	0	277	26	35	22	114	1	8	37	196	33	+ 0	25	66	19
21	20	271	49	43	20	118	6	17	2	194	40	9	19	57	10
21	40	263	21	51	51	123	7	26	26	192	23	19	19	46	51
22	0	247	33	60	59	130	20	37	42	189	2	31	29	34	15
− 22	20	202	9	+ 68	1	147	35	+ 54	33	180	40	+ 50	42	14	0

EXPLANATION OF THE CHARTS AND THEIR USE.

These charts are the development, by the stereographic projection, of a sphere one foot in diameter The center of each chart is in latitude 22° 54′ south, and the border is at the distance of 100° from this zenith The charts are so placed in longitude that the illuminated hemisphere, at the time of contact, may occupy the central portion of the chart The longitudes, which are noted along the equator, are counted westerly from the meridian of Washington, and, to avoid the inconvenience of repeating the same figures as eastern longitudes, which might give rise to provoking errors in the use of these charts, the numeration has been carried beyond 180° up to 360° The latitudes are noted along the middle meridian The smaller islands of the Pacific and Indian Oceans have been indicated on the charts only in those regions which are favorably situated for observations for determining the parallax, and no geographical names have been placed on the charts, except the names of a few principal cities and towns and islands in the same regions

As to the curves delineated on the charts, one will notice, first, the continuous line in black limiting all the other broken lines, and on which is inscribed "Contact on the horizon" At all points on this line the phenomenon of contact, mentioned in the title of the chart, will take place in the horizon, that is, the point of contact common to the limbs of Venus and the sun will be in the horizon As stated above, the horizontal refraction has been allowed for in determining the position of this curve This curve, then, limits the part of the earth's surface in which the phenomenon is visible At all points on the curve to the right of the middle meridian of the chart, the sun will be setting at the time of the phenomenon, and at all points to the left it will be rising

At any station within this curve, the altitude of Venus and the sun will be approximately equal to the arc of a great circle drawn from the station to meet the curve at right angles On each chart will be found a "Scale for altitudes" If, having plotted any station on the chart, we measure, with a pair of dividers, the shortest distance to the curve of "Contact on the horizon," and then apply this distance to the scale, we shall get the altitude of the point of contact, with an error of not more than $\frac{1}{4}$°, or, in extreme cases, $\frac{1}{2}$°, an approximation sufficient for the purpose of estimating the extinction of light The scale is limited to 30°, as, beyond this point, the extinction of light is not of importance

The azimuth of the point of contact, differing but little from that of the sun's center, may be estimated from the chart by drawing a line from the station to the central point of the chart, which lies on the middle meridian in latitude 22° 54′ south, and measuring the angle which this line makes with the meridian of the station To avoid drawing this meridian, we may take points in the same latitude on the two nearest meridians, find the azimuth at each, and then, by interpolation, the azimuth at the station

One will notice, next, the dotted lines in blue At all points, on each of these, the phenomenon of contact takes place at the same instant, the corresponding Washington mean time of which is noted at the right-hand extremity of the line It will be noticed that there is an interval of one minute between the times corresponding to the first five and last five curves on the chart, but that elsewhere the interval is two minutes In addition to these curves, the positions, situated on the curve of "Contact on the hor-

izon," where the phenomenon of contact makes its first and last appearance on the earth's surface, are indicated, as are also the times of their occurrence

In the region crossed by the curves passing through the central part of the chart, where the intervals between the successive curves are nearly equal, one will have no difficulty in interpolating between them Having plotted the station on the chart by its given longitude, counted west from Washington, and its latitude, conceive a line to be drawn through it, perpendicular to each of the adjacent curves, and, having ascertained the proportion of the parts into which the station divides this line, find the time which divides the interval between the times belonging to the adjacent curves in the same ratio This will be the Washington mean time of the contact at the station Subtracting from this the longitude west from Washington, converted into time, one will get the local mean time of contact It may be well to notice here that in Eastern Europe, Asia, Africa, Australia, and New Zealand, where the time has been arrived at in going eastward from Europe, the transit occurs, in civil time, on December 9, but that in the Sandwich and other islands of the Pacific, where the time has been arrived at in going westward from America and Europe, the transit occurs on December 8.

In the regions near the points of first and last appearance of contact, interpolation between the curves is more difficult, owing to the irregularity of the intervals A satisfactory result, however, can be obtained from using the principle that the interval between the times, corresponding to two time-curves, is nearly proportional to the difference of the cosines of the maximum altitudes, at which contact occurs on these curves This is best illustrated by an example Let it be required to find the time of interior contact at egress, at Khiva, in the region east of the Caspian Sea On referring to chart No 3, the time is seen to lie between $12^h\ 59^m$ and $13^h\ 0^m$ From the tables given above, we find that the maximum altitudes at which contact occurs on these time-curves are, respectively, $26°\ 50'$ and $11°\ 45'$ Interpolating between these, as in the preceding case, we shall find that the maximum altitude of the time-curve which passes through Khiva is about $23°\ 8'$ Then the required time is given by the following expression

$$12^h\ 59^m + \frac{\cos 23°\ 8' - \cos 26°\ 50'}{\cos 11°\ 45' - \cos 26°\ 50'} \times 1^m$$

and this is $12^h\ 59^m\ 19^s$ When the station at which it is desired to find the time of contact lies within the first or last time-curve drawn on the chart, the point of first or last appearance of contact on the earth's surface, with its associated time, takes the place in the interpolation of one of the time-curves The error of the time of contact, derived in this way, ought not to exceed 5^s However, for stations in the central portions of the chart, it may sometimes be a little more In this error we must be understood as including only errors of drawing, plotting, and measuring upon the chart, and not errors in the elements, on which the computations for the charts have been based, the effect of which may be very much larger

One will notice, lastly, the dotted lines in red At all points, on each of these, contact occurs at the same point on the sun's limb The angle of position of this point, counted from the north point of the limb toward the east, for the two charts which belong to the ingress, but toward the west for the two which belong to the egress, is noted at the left-hand extremity of each curve The interval between the

angles, corresponding to two adjacent curves, is uniformly 20′ The angle of position of the point of contact, for any station, can be found in precisely the same manner as the time from the time-curves The greatest and least angles of position of the point of contact which occur on the curve of "Contact on the horizon" are not noted on the charts As they may be needed in interpolation, I give them here

	Minimum value	Maximum value
	° ′	° ′
Chart No 1	40 55 6	51 49 1
Chart No 2	39 58 4	46 13 6
Chart No 3	10 33 1	16 18 3
Chart No 4	17 30 3	22 23 8

TABLES AND FORMULÆ FOR COMPUTING TIMES OF CONTACT

As more accurate values of the times of contact may be desired than can be derived from the charts, tables of data, entirely similar to the data for solar eclipses given in the American Ephemeris, are here appended

I —*Exterior Contact at Ingress*

Wash M T	*A*	*B*	*C*	*μ*
h m				° ′
8 29	+ 32 9620	+ 67 4082	− 14 8430	129 14 7
30	32 7995	67 4513	14 7999	129 29 7
31	32 6370	67 4944	14 7568	129 44 6
32	32 4745	67 5375	14 7137	129 59 6
33	32 3119	67 5806	14 6706	130 14 6
34	32 1494	67 6237	14 6275	130 29 5
35	31 9869	67 6667	14 5845	130 44 5
36	31 8243	67 7098	14 5414	130 59 5
37	31 6618	67 7529	14 4983	131 14 5
38	31 4993	67 7960	14 4552	131 29 4
39	31 3368	67 8390	14 4122	131 44 4
40	31 1743	67 8821	14 3691	131 59 4
41	31 0118	67 9251	14 3261	132 14 3
42	30 8493	67 9682	14 2830	132 29 3
43	30 6867	68 0113	14 2399	132 44 3
44	30 5242	68 0543	14 1969	132 59 2
45	30 3617	68 0974	14 1538	133 14 2
46	30 1992	68 1404	14 1108	133 29 2
47	30 0367	68 1835	14 0677	133 44 1
48	29 8741	68 2265	14 0247	133 59 1
49	29 7116	68 2696	13 9816	134 14 1
50	29 5491	68 3126	13 9386	134 29 0
8 51	+ 29 3866	+ 68 3557	− 13 8955	134 44 0

II —*Interior Contact at Ingress*

Wash M T		*A*	*B*	*C*	μ		
h	m				°	′	
8	57	+ 28 4115	+ 65 9731	− 10 9967	136	13	8
	58	28 2490	66 0162	10 9536	136	28	8
	59	28 0865	66 0592	10 9106	136	43	8
9	0	27 9239	66 1022	10 8676	136	58	7
	1	27 7614	66 1452	10 8246	137	13	7
	2	27 5988	66 1882	10 7816	137	28	7
	3	27 4363	66 2312	10 7386	137	43	6
	4	27 2737	66 2742	10 6956	137	58	6
	5	27 1112	66 3172	10 6526	138	13	6
	6	26 9487	66 3602	10 6096	138	28	6
	7	26 7861	66 4032	10 5666	138	43	5
	8	26 6236	66 4462	10 5236	138	58	5
	9	26 4610	66 4892	10 4806	139	13	5
	10	26 2985	66 5322	10 4376	139	28	4
	11	26 1360	66 5752	10 3946	139	43	4
	12	25 9734	66 6182	10 3516	139	58	4
	13	25 8108	66 6611	10 3087	140	13	3
	14	25 6482	66 7041	10 2657	140	28	3
	15	25 4857	66 7471	10 2227	140	43	3
	16	25 3232	66 7901	10 1797	140	58	2
	17	25 1606	66 8330	10 1368	141	13	2
	18	24 9981	66 8760	10 0938	141	28	2
	19	24 8355	66 9189	10 0509	141	43	1
	20	24 6730	66 9619	10 0079	141	58	1
	21	24 5104	67 0019	9 9649	142	13	1
	22	24 3479	67 0478	9 9220	142	28	1
9	23	+ 24 1853	+ 67 0908	− 9 8790	142	43	0

III —*Interior Contact at Egress*

Wash M T		*A*	*B*	*C*	μ
h	m				°
12	35	− 7 0414	+ 75 2908	− 1 6806	190 37 0
	36	7 2041	75 3332	1 6382	190 52 0
	37	7 3668	75 3757	1 5957	191 7 0
	38	7 5296	75 4181	1 5533	191 22 0
	39	7 6923	75 4606	1 5108	191 36 9
	40	7 8550	75 5030	1 4684	191 51 9
	41	8 0177	75 5454	1 4260	192 6 9
	42	8 1804	75 5879	1 3835	192 21 8
	43	8 3432	75 6303	1 3411	192 36 8
	44	8 5059	75 6728	1 2986	192 51 8
	45	8 6686	75 7152	1 2562	193 6 7
	46	8 8313	75 7576	1 2138	193 21 7
	47	8 9941	75 8000	1 1714	193 36 7
	48	9 1568	75 8425	1 1289	193 51 6
	49	9 3196	75 8849	1 0865	194 6 6
	50	9 4823	75 9273	1 0441	194 21 6
	51	9 6450	75 9697	1 0017	194 36 6
	52	9 8078	76 0121	0 9593	194 51 5
	53	9 9705	76 0546	0 9168	195 6 5
	54	10 1333	76 0970	0 8744	195 21 5
	55	10 2960	76 1394	0 8320	195 36 4
	56	10 4587	76 1818	0 7896	195 51 4
	57	10 6215	76 2242	0 7472	196 6 4
	58	10 7842	76 2665	0 7049	196 21 3
	59	10 9470	76 3089	0 6625	196 36 3
13	0	11 1097	76 3513	0 6201	196 51 3
13	1	− 11 2724	+ 76 3937	− 0 5777	197 6 2

IV.—Exterior Contact at Egress

Wash M T	A	B	C	μ
h m				° ′
13 7	− 12 2489	+ 79 2889	− 2 9645	198 36 1
8	12 4117	79 3313	2 9221	198 51 0
9	12 5744	79 3736	2 8798	199 6 0
10	12 7372	79 4160	2 8374	199 21 0
11	12 9000	79 4584	2 7950	199 35 9
12	13 0627	79 5007	2 7527	199 50 9
13	13 2255	79 5431	2 7103	200 5 9
14	13 3882	79 5854	2 6680	200 20 8
15	13 5510	79 6278	2 6256	200 35 8
16	13 7138	79 6701	2 5833	200 50 8
17	13 8765	79 7125	2 5409	201 5 7
18	14 0393	79 7548	2 4986	201 20 7
19	14 2020	79 7972	2 4563	201 35 7
20	14 3648	79 8395	2 4139	201 50 6
21	14 5276	79 8818	2 3716	202 5 6
22	14 6903	79 9242	2 3292	202 20 6
23	14 8531	79 9665	2 2869	202 35 6
24	15 0158	80 0089	2 2445	202 50 5
25	15 1786	80 0512	2 2022	203 5 5
26	15 3414	80 0935	2 1599	203 20 5
27	15 5041	80 1358	2 1176	203 35 1
28	15 6669	80 1782	2 0752	203 50 4
13 29	− 15 8296	+ 80 2205	− 2 0329	204 5 4

The other quantities needed for the computation may be taken to be constant for each contact, and have the following values

	log E	log F	log G	log H	A'	B' and C'
I	9 96323	9 96562	n 9 59632	n 9 58290	− 27 087	+ 7 177
II	9 96323	9 96558	n 9 59629	n 9 58312	− 27 091	+ 7 161
III	9 96311	9 96546	n 9 59697	n 9 58382	− 27 122	+ 7 070
IV	9 96307	9 96546	n 9 59718	n 9 58379	− 27 127	+ 7 057

A', B', and C' are respectively the variations of A, B, and C in one second, and are expressed in units of the fourth decimal place

INVESTIGATION

OF

CORRECTIONS TO HANSEN'S TABLES OF THE MOON;

WITH

TABLES FOR THEIR APPLICATION.

BY

SIMON NEWCOMB,

PROFESSOR, U S NAVY

FORMING PART III OF PAPERS PUBLISHED BY THE COMMISSION ON THE TRANSIT OF VENUS

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INTRODUCTORY NOTE.

When the problem of utilizing the observations of occultations at the several Transit of Venus stations, so as to determine the longitudes of those stations with all attainable accuracy, was presented to the Commission on the Transit of Venus, it was found necessary to make a careful determination of the errors of the lunar ephemeris before an entirely satisfactory solution of the problem could be attempted. The Secretary of the Commission was therefore charged with this work, most of the computations on which have been made under his direction by Mr. D. P. Todd, computer for the Commission.

WASHINGTON, *May* 25, 1876.

CORRECTIONS TO BE APPLIED TO HANSEN'S TABLES OF THE MOON

§ 1

INVESTIGATION OF ERRORS OF LONGITUDE

One of the most important operations in connection with the observations of the transit of Venus is the accurate determination of the longitudes of the stations Many of these stations are so far removed from telegraphic communication that the longitudes must depend mainly on the moon Determinations of longitude from moon culminations are found by experience to be subject to constant errors which it is difficult to determine and allow for It was therefore a part of the policy of the American Commission to depend on occultations rather than upon moon culminations for the determination of longitudes The reason for this course is, that the disappearance of a star behind the limb of the moon is a sudden phenomenon, the time of which can always be fixed within a fraction of a second If the ephemeris of the moon and star were correct, and the disk of the former a perfect circle, the longitude could be determined from the occultation with the same degree of accuracy that the phenomenon could be observed The question arises, how far these sources of error can be diminished The inequalities of the lunar surface form a source of error which it is impossible to avoid, but which is comparatively innocuous when many observations are made, since the errors will be purely accidental, and will therefore be eliminated from the mean of a great number of observations

The position of the star can be determined by meridian observations with almost any required degree of accuracy We have, then, only to see how far the errors of the lunar ephemeris can be diminished, and to reduce these errors to a minimum is the object of the present paper

Hansen's tables are taken for this purpose, because there is reason to believe that the perturbations on which these tables are founded are, in the main, extremely accurate, more accurate and complete, in fact, than any others which have been tabulated Still, before they can be used for the purpose in question, a number of very important corrections are required, which we may divide into two classes,—corrections to the theory, and to the elements

It is well known that Hansen increased all the perturbations of his tables by the constant factor 0.0001544, on account of a supposed want of coincidence between the

center of figure and the center of gravity of the moon. I have shown that Hansen fails to sustain this position, and that there is no good reason to suppose that the moon differs from any other of the heavenly bodies in this respect.* Our first course would therefore be to diminish all of Hansen's inequalities by this factor, were it not that there are reasons why each of the two greatest perturbations of the moon's motion,—the evection and the variation,—should be found larger from observation than he found them from theory.

Evection.—The evection has the eccentricity as a factor, the value of the other factor being nearly 0.4. If, then, the adopted eccentricity of the moon be erroneous, the computed evection will be erroneous by four-tenths the amount of the error. Now, by reference to Hansen's "*Darlegung der theoretischen Berechnung der in den Mondtafeln angewandten Störungen*"† (page 173), it will be seen that the eccentricity adopted throughout in the computation of the perturbations of the moon is less by 0.0000073 than the value he finally found from observation, and adopted in the tables. Had he used the latter value, the theoretical evection would have been greater by the fraction $\frac{.0000073}{.0549008} = 0.000133$. The factor actually used being 0.0001544, the evection, thus increased, is too large by only 0.000021 of its entire amount, or 0″.09. Consequently, the tabular coefficient of evection should be diminished by this amount. Precisely the same result follows, if we adopt Hansen's view of a separation of the centers of figure and gravity of the moon, and Hansen himself is led to it on page 175 of the work cited, only instead of 0″.09, he says, "kein volles Zehntheil einer Secunde."

Variation.—That the coefficient of variation resulting from meridian observations will be greater than the actual coefficient may be anticipated from the following considerations. The inequality in question attains its maxima and minima in the moon's octants. In the first octant, we have a maximum. The elongation of the moon from the sun is then about 3^h; and the observed position of the moon is mainly dependent on observations of the first limb made in the daytime, when the apparent semi-diameter of the moon will be diminished by the brilliancy of the surrounding sky. No account of this diminution of the apparent semi-diameter being taken in the reductions, the semi-diameter actually applied is too large, and the observed right ascension of the moon is also too large.

When the moon reaches the third octant, the value of the variation attains its minimum. The moon then transits at 9^h, and the meridian observation is made on the first limb, while the apparent semi-diameter is increased by the irradiation consequent upon the contrast between the moon and the sky. The result will be that the observed right ascension will be too small.

The same causes will make the observed right ascension too great in the fifth octant, and too small in the seventh. These positive and negative errors of observed right ascension correspond to the times of maximum and minimum effects of variation in increasing the longitude of the moon. Therefore, the observed variation will appa-

* Proceedings of the American Association for the Advancement of Science, 1868.—Silliman's American Journal of Science, November, 1868.

† Abhandlungen der mathematisch-physischen Classe der Königlich Sächsischen Gesellschaft der Wissenschaften Band vi.

rently be larger than the actual variation, whatever this may be This seems a much more natural and probable cause for the apparent excess of the observed over the theoretical perturbations than that assigned by Hansen Hansen's factor only increases the coefficient in question by 0″ 33, but it seems probable that the variation derived from observations alone would be yet larger than Hansen's increased variation. In fact, in 1867, I found, by comparing the errors of the lunar ephemeris when the moon culminated at different times of the day, that the effect of the greater irradiation at night was very strongly marked During the four years 1862–65 the mean errors of the tables in right ascension at different times of day were as follows *

	s.
Before sunset	−0 154
After bright daylight in the evening	−0 093
Before bright daylight in the morning	+0 091
After sunrise	+0 153

In the difference between the results for each limb, the effect of increased irradiation seems to be 0^s06

The only remaining term which is large enough to be materially affected by the increase in question is the annual equation, of which the increase is 0″ 10

A glance at the errors of Hansen's tables, given by meridian observations, will show that the errors about the time of first quarter, and, indeed, during the first half of the lunation, are in the mean less by between 3″ and 4″ than during the second half Hence, either the semi-diameter, or the parallactic equation, or both, are too large The parallactic equation used by Hansen corresponds to a value 8″ 916 for the solar parallax, which value is too large by probably not much less than 0″ 10 The result which I deduced in 1867 from all the really valuable data extant was 8″ 848, and the determinations which have since been made, when revised with the best data, seem to indicate a diminution of this value rather than an increase These indications are, however, as yet, a little too indefinite to predicate anything upon I shall therefore continue to use 8″ 848, which will diminish Hansen's value by 0″.068 The corresponding diminution in the principal parallactic term will be 0″ 96, while there will be two other terms to receive a smaller diminution

This correction will still leave a difference of about 2″ between the results from the first and second limbs, which will be accounted for by an error of 1″ in the adopted semi-diameter This correction to the semi-diameter is *a priori* quite probable, as the improved meridian instruments of the present time give a semi-diameter of the sun 1″ less than the older ones from which the diameters adopted in our ephemerides were derived It is to be expected that the semi-diameter of the moon will exhibit a similar apparent diminution

From a note in Hansen's *Darlegung* (page 439), it will be seen that one of the terms in the true longitude has crept into the tables with a wrong sign As employed in the tables, and given on page 15 of the introduction, it is, $+0'' 335 \sin(2g - 4g' + 2\omega - 4\omega')$

As revised in the *Darlegung*, it is $-0'' 285 \sin$

Therefore the tables need the correction $-0'' 62 \sin$

* Investigation of the Distance of the Sun, p 24

The following is a list of the corrections we have so far deduced to Hansen's tables. They should in strictness be applied to the mean longitude, or "*Argument fondamental*", but they may without serious error be applied to the true longitude.

Put

- D, the argument of parallactic inequality, or mean elongation of the moon from the sun;
- g, the moon's mean anomaly;
- g', the sun's mean anomaly;
- ω, the distance of the moon's perigee from the ascending node;
- ω', the distance of the sun's perigee from the same node.

We then have

$$D = g - g' + \omega - \omega',$$

and the corrections in question are,

$$\left.\begin{array}{l} +0''.96 \sin D \\ +0.07 \sin (D - g) \\ -0.13 \sin (D + g') \end{array}\right\} \textit{Parallactic terms.}$$

$$+0.09 \sin g' \qquad \textit{Annual equation.}$$

$$-0.33 \sin 2D \qquad \textit{Variation.}$$

$$-0.10 \sin (2D - g) \qquad \textit{Evection.}$$

$$-0.62 \sin (g\,2 - 4g' + 2\omega - 4\omega') \qquad \textit{Accidental error.}$$

The fourth and fifth terms of this expression have the effect to remove the increase which Hansen applied to his inequalities on account of the position of the center of gravity of the moon, while the sixth is the result of the slight error of the eccentricity which he employed in computing the coefficient of evection.

In comparing with meridian observations which have been reduced without any correction to the apparent semi-diameter depending on the time of day, the correction of variation may also be omitted, since a yet larger apparent correction, having the opposite algebraic sign, will result from the apparent variations of that semi-diameter, as already explained.

As regards the possible corrections to the elements of Hansen's tables, it is to be remarked that that investigator did not avail himself of the elements of the lunar orbit deduced by Airy from the Greenwich observations between 1750 and 1830, but obtained his final values of the elements by a comparison of his own. Of the nature and extent of the observations thus employed, we have no details; but it is not likely that more than a very small fraction of the entire mass of observations was used, and it can therefore hardly be expected that the elements were determined with the last degree of accuracy. Any error in the motion of the perigee or node will constantly increase with the time. If, in addition to this, we reflect that the meridian observations of the last twenty years are far more accurate than those Hansen had at his disposal, it will not seem at all surprising to find quite sensible errors in the present longitudes of the lunar perigee and node as derived by Hansen. Our next step will therefore be to determine

what corrections to Hansen's elements are indicated by the recent observations of the moon made at Greenwich and Washington since 1862, a period during which both series of observations are carefully compared with Hansen's tables

The general ideas on which the present investigation of these corrections is based are these the errors of the moon's tabular longitude are of two classes,—a progressive correction, which apparently increases uniformly with the time, and errors of short period, the principal ones of which go through their period during one revolution of the moon or less In determining the errors of the first class from observation, those of the second class may be regarded as accidental errors, the effect of which will be eliminated from the mean of a large number of observations Since, in a series of observations extending through a number of years, the maxima and minima of each term of short period will fall indiscriminately into all parts of all the other periods, each periodic correction may be determined as if the effects of the others were purely accidental errors At the same time, as the elimination of each periodic error from the maxima and minima of all the others cannot be complete in any finite time, it is desirable that each periodic correction of sensible magnitude which we can determine beforehand shall be applied to the residuals before the latter are used to determine the corrections to the elements

The corrections of the elements of longitude have been made to depend principally upon the observed right ascensions, instead of reducing the observed errors of right ascension and polar distance to errors of longitude and latitude The reason for this course is, that the apparent errors of polar distance, after correcting them approximately for errors of the elements easily determined, will arise principally from errors of observation, and not from errors of the tables In fact, the observations of the moon's declination are sometimes affected with accidental errors of a magnitude which it is difficult to account for, especially in the case of Washington Granting that the moon moves in a plane the position of which can be very accurately determined, we have afterward only to determine the moon's position in that plane, and this can be done from an observed right ascension almost as well as if we had a directly observed longitude The longitude thus determined will be less likely to be affected with systematic errors than if we suppose the position entirely unknown, and change the errors of right ascension and declination to errors of longitude and latitude, without regard to the possible constant errors of the measured declinations

Formulæ for expressing the longitude and latitude of the moon in terms of the lunar elements are given by Hansen in a posthumous memoir* The following terms are sufficient for our present purpose

Put

l, the moon's longitude in orbit,

θ, the longitude of the ascending node,

i, the inclination of the orbit to the ecliptic,

α, δ, the moon's right ascension and declination,

ω, the obliquity of the ecliptic

* Ueber die Darstellung der graden Aufsteigung und Abweichung des Mondes in Function der Länge in der Bahn und der Knotenlänge Abhandlungen der Königlich-Sächsischen Gesellschaft der Wissenschaften, Bd x, No viii

We then have, approximately,

$$\alpha = l - 2^\circ.5 \sin 2\,l - 1^\circ.1 \sin (2\,l - \theta) + 1^\circ.1 \sin \theta$$

$$\sin \delta = \sin \omega \sin l + \cos \omega \sin i \sin (l - \theta)$$

$$= 0.40 \sin l + 0.08 \sin (l - \theta)$$

The differential co-efficients derived from these expressions are,

$$\frac{d\alpha}{dl} = 1 - 0.037 \cos (2\,l - \theta) - 0.087 \cos 2\,l$$

$$\frac{d\alpha}{d\theta} = 0.018 \cos \theta + 0.018 \cos (2\,l - \theta)$$

$$\frac{d\alpha}{di} = 0.21 \sin \theta - 0.21 \sin (2\,l - \theta)$$

$$\cos \delta \frac{d\delta}{dl} = 0.40 \cos l + 0.08 \cos (l - \theta)$$

$$= (0.40 + 0.08 \cos \theta) \cos l + 0.08 \sin \theta \sin l$$

$$\cos \delta \frac{d\delta}{d\theta} = - 0.081 \cos (l - \theta)$$

$$\cos \delta \frac{d\delta}{di} = 0.92 \sin (l - \theta)$$

From the first three formulæ, it will be seen, that the mean error in right ascension is very nearly the same as the mean error in longitude; the periodic corrections being supposed to be eliminated from this mean.

The investigation of the corrections from observations is now made as follows: All the apparent errors of the tables derived from the meridian observations at Greenwich and Washington since 1862 have been collected, arranged in the order of dates and the mean taken for each year; observations of the separate limbs being kept separate. The mean error in right ascension for each year is as follows:

Apparent errors of Hansen's tables in R. A.

	Greenwich.			Washington.			Mean.		
Year.	I.	II.	Diff.	I.	II.	Diff.	I.	II.	Mean.
	″	″	″	″	″	″	″	″	″
1862	. .	. .	. .	. .	. .	. .	− 3.6	− 0.6	− 2.1
1863	. .	. .	. .	. .	. .	. .	− 2.3	+ 0.5	− 0.9
1864	. .	. .	. .	. .	. .	. .	− 1.0	+ 1.8	+ 0.4
1865	− 0.2	+ 3.0	3.2	+ 0.3	+ 3.9	3.6	0.0	+ 3.4	+ 1.7
1866	+ 1.2	+ 3.6	2.4	+ 0.9	+ 4.5	3.6	+ 1.0	+ 4.0	+ 2.5
1867	+ 2.4	+ 5.7	3.3	+ 2.4	+ 5.8	3.4	+ 2.4	+ 5.8	+ 4.1
1868	+ 2.6	+ 6.0	3.4	+ 2.4	+ 6.6	4.2	+ 2.5	+ 6.3	+ 4.4
1869	+ 3.3	+ 5.6	2.3	+ [illegible]	+ 7.4	4.0	+ 3.4	+ 6.5	+ 4.9
1870	+ 3.4	+ 6.6	3.2	+ 4.6	+ 7.2	2.6	+ 4.0	+ 6.9	+ 5.4
1871	+ 5.4	+ 8.2	2.8	+ 5.1	+ 7.8	2.7	+ 5.2	+ 8.0	+ 6.6
1872	+ 6.0	+ 8.7	2.7	+ 6.2	+ 9.6	3.4	+ 6.1	+ 9.2	+ 7.6
1873	+ 6.9	+ 9.4	2.5	+ 6.9	+10.2	3.3	+ 6.9	+10.2	+ 8.6
1874	+ 8.1	+11.4	3.3	+ 7.1	+10.8	3.7	+ 7.6	+11.1	+ 9.4

The last column exhibits the apparent tabular errors in mean right ascension, and

therefore in mean longitude, as derived each year from all the observations. The sudden apparent alteration of nearly one second per annum in the mean motion of the moon, exhibited in this column, seems to me one of the most extraordinary of astronomical phenomena; but, as I have discussed it in several papers during the last five years, I shall do no more here than call attention to its continuance, and to the impossibility of representing it by any small number of periodic terms without introducing discordances into the longitude during previous years.

It will be seen that there are discordances between the results of the two observatories, sometimes amounting to more than a second. In determining the corrections of short period, it is desirable to reduce the systematic errors extending through each year to a minimum, the question whether such errors are in the theory or the observations being indifferent. It is also desirable that in taking the mean of the results of the two observatories, they should be made comparable with each other by correcting either of them for the systematic difference. These corrections, of course, only admit of approximate determination, and they have been applied each year to that observatory or that limb of the moon in which, judging from the deviations from uniform progression, it was judged most likely that the discordance existed. The following are the corrections actually applied to the several classes of tabular errors.

Year	Greenwich I	Greenwich II	Washington I	Washington II
	s	s	s	s
1862–63	+0.06	+0.06	0	0.00
1864	0	0	0	0
1865–68	0	0	0	−0.04
1869	0	+0.06	0	−0.04
1870	+0.06	0	0	−0.04
1871	0	0	0	0
1872	0	0	0	−0.04
1873–74	0	0	0	0

Having applied these corrections throughout their several years, the Greenwich and Washington observations were considered strictly comparable, and when the moon was observed at both observatories on the same day, the mean of the corrected tabular errors was taken. The mean outstanding tabular error for each year now becomes as follows:

Year	$\delta\lambda$	Year	$\delta\lambda$	Year	$\delta\lambda$	Year	$\delta\lambda$
	″		″		″		″
1862	−2.1	1866	+2.2	1869	+5.1	1872	+7.3
1863	−0.9	1867	+3.8	1870	+5.6	1873	+8.6
1864	+0.4	1868	+4.1	1871	+6.6	1874	+9.7
1865	+1.4						

These quantities, with the sign changed, should be considered as corrections to the fundamental argument, and we have to determine the corresponding correction to the right ascensions which are to be applied to the individual tabular errors. To reduce them to corrections of true longitude, they are to be multiplied by the factor

$$1 + 2e \cos g = 1 + 0.11 \cos g$$

The corresponding factor for correction of right ascension is, with sufficient approximation,

$$\delta\alpha = (1 + 0.11 \cos g - 0.04 \cos(2l - \theta) - 0.09 \cos 2l)\, \delta\lambda$$

In this formula, $\delta\lambda$ represents the correction to the mean longitude, while we may suppose l to represent indifferently the mean or the true longitude; and, during a period of several months at a time, we may represent the longitude as a function of g. The value of $\delta\alpha$ has been reduced to a table of double entry as a function of g and of the time. To express the mean longitude as a function of g, we have

$$\begin{aligned} l &= g + \pi \\ 2l - \theta &= 2g + 2\pi - \theta \\ 2l &= 2g + 2\pi \end{aligned}$$

By the substitution of these values, the expression for $\delta\alpha$ becomes

$$\delta\alpha = (1 + 0.11 \cos g + A \cos 2g + B \sin 2g)\, \delta\lambda$$

where

$$\begin{aligned} A &= -.04 \cos(2\pi - \theta) - .09 \cos 2\pi \\ B &= .04 \sin(2\pi - \theta) + .09 \sin 2\pi \end{aligned}$$

The values of π, θ, A, and B for periods of six months are as follow:

Year.	π	θ	A	B	Year.	π	θ	A	B
	°	°				°	°		
1862.0	228	274	+ .05	+ .09	1869.0	153	139	− .01	− .06
1862.5	248	264	+ .09	+ .09	1869.5	173	129	− .07	− .05
1863.0	269	255	+ .08	− .04	1870.0	194	119	− .08	.00
1863.5	289	245	+ .03	− .08	1870.5	214	110	− .06	+ .05
1864.0	309	235	− .02	− .07	1871.0	234	100	− .01	+ .09
1864.5	330	226	− .05	− .04	1871.5	255	90	+ .06	+ .08
1865.0	350	216	− .06	.00	1872.0	275	81	+ .10	+ .02
1865.5	310	206	− .05	+ .03	1872.5	295	71	+ .09	− .06
1866.0	31	197	− .01	+ .08	1873.0	316	61	+ .04	− .11
1866.5	51	187	+ .02	+ .05	1873.5	336	52	− .05	− .11
1867.0	71	177	+ .05	+ .03	1874.0	356	42	− .12	− .04
1867.5	92	168	+ .05	.00	1874.5	17	32	− .12	+ .05
1868.0	112	158	+ .04	− .02	1875.0	37	23	− .04	+ .12
1868.5	133	148	+ .03	− .05					

The coefficient $1 + 0.11 \cos g + A \cos 2g + B \sin 2g$ is next tabulated for each of these sets of values of A and B for every 10° of g, and multiplied by the corresponding value of $\delta\lambda$. As these tables are superseded by those given at the close of this paper, it is not necessary to print them.

The corrections of short period, which have been actually applied, are

$$\begin{aligned} &+0''.96 \sin D \\ &-0.13 \sin(D + g') \\ &+0.09 \sin g' \\ &-0.62 \sin(2g - 4g' + 2\omega - 4\omega') \end{aligned}$$

The first three have been combined into a single one of double argument, in which the arguments are D and the month, the latter corresponding to g'. The terms dependent on this argument are so small that they may be regarded as constant during an entire month

In this same table is included a partially conjectural correction for the variations of the moon's semi-diameter The correction to Hansen's value has been assumed as $-2'' 0$, when the moon is in the neighborhood of the sun, so that her limb is very faint, and as $-0'' 4$ after the close of evening twilight Between two hours of elongation and the close of twilight, it is assumed to increase uniformly The sum of these four corrections is given in the following table

Days before mean full moon	FIRST LIMB												Days before mean full moon
	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec	
	"	"	"	"	"	"	"	"	"	"	"	"	
− 14	+2 4	+2 5	+2 5	+2 6	+2 5	+2 4	+2 3	+2 2	+2 1	+2 1	+2 2	+2 3	− 14
− 13	+2 3	+2 4	+2 5	+2 5	+2 4	+2 3	+2 2	+2 1	+2 0	+2 0	+2 1	+2 2	− 13
− 12	+2 2	+2 3	+2 5	+2 4	+2 4	+2 3	+2 2	+2 1	+2 0	+1 9	+2 0	+2 0	− 12
− 11	+2 1	+2 2	+2 4	+2 4	+2 3	+2 2	+2 1	+2 1	+2 0	+1 8	+1 9	+2 0	11
− 10	+2 0	+2 1	+2 4	+2 3	+2 2	+2 1	+2 0	+2 0	+2 0	+1 7	+1 7	+1 8	10
− 9	+1 8	+2 0	+2 3	+2 2	+2 1	+2 1	+2 0	+1 9	+1 8	+1 6	+1 5	+1 6	− 9
− 8	+1 5	+1 7	+2 1	+2 0	+2 0	+2 0	+1 9	+1 8	+1 7	+1 4	+1 4	+1 5	− 8
− 7	+1 5	+1 5	+1 8	+1 8	+1 8	+1 8	+1 7	+1 6	+1 4	+1 3	+1 4	+1 5	− 7
− 6	+1 5	+1 4	+1 4	+1 5	+1 6	+1 5	+1 5	+1 4	+1 2	+1 3	+1 4	+1 4	− 6
− 5	+1 4	+1 3	+1 3	+1 2	+1 4	+1 4	+1 3	+1 1	+1 2	+1 2	+1 3	+1 4	− 5
− 4	+1 2	+1 2	+1 2	+1 1	+1 1	+1 1	+1 0	+1 1	+1 1	+1 1	+1 2	+1 2	− 4
− 3	+1 1	+1 0	+1 0	+1 0	+0 9	+0 9	+0 9	+1 0	+1 0	+1 0	+1 1	+1 1	− 3
− 2	+0 9	+0 8	+0 8	+0 8	+0 7	+0 7	+0 8	+0 8	+0 8	+0 8	+0 9	+0 9	2
− 1	+0 6	+0 6	+0 6	+0 6	+0 6	+0 6	+0 6	+0 6	+0 6	+0 6	+0 7	+0 6	− 1
0	+0 4	+0 4	+0 4	+0 4	+0 4	+0 4	+0 4	+0 4	+0 4	+0 4	+0 4	+0 4	0

Days after mean full moon	SECOND LIMB												Days after mean full moon
	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec	
	"	"	"	"	"	"	"	"	"	"	"	"	
0	−0 4	−0 4	−0 4	−0 4	−0 4	−0 4	−0 4	−0 4	−0 4	−0 4	−0 4	−0 4	0
+ 1	−0 6	−0 6	−0 6	−0 6	−0 6	−0 6	−0 6	−0 6	−0 6	−0 6	−0 6	−0 6	+ 1
+ 2	−0 8	−0 7	−0 8	−0 8	−0 8	−0 8	−0 7	−0 7	−0 8	−0 8	−0 8	−0 8	+ 2
+ 3	−1 1	−0 9	−1 0	−1 0	−0 9	−0 9	−0 9	−0 9	−1 0	−1 0	−1 0	−1 1	+ 3
+ 4	−1 2	−1 1	−1 2	−1 1	−1 1	−1 0	−1 0	−1 1	−1 1	−1 2	−1 2	−1 2	+ 4
+ 5	−1 4	−1 2	−1 2	−1 2	−1.4	−1 4	−1 4	−1 2	−1 3	−1 3	−1 4	−1 4	+ 5
+ 6	−1 4	−1 3	−1 3	−1 4	−1 5	−1 5	−1 5	−1 5	−1 4	−1 4	−1 4	−1 5	+ 6
+ 7	−1 5	−1 3	−1 6	−1 6	−1 7	−1 7	−1 8	−1 7	−1 6	−1 5	−1 5	−1 5	+ 7
+ 8	−1 4	−1 5	−1 8	−1 8	−1 9	−1 9	−2 0	−1 9	−1 9	−1 7	−1 5	−1 5	+ 8
+ 9	−1 7	−1 7	−2 0	−1 9	−1 9	−2 0	−2 1	−2 1	−2 1	−1 8	−1 8	−1 7	+ 9
+ 10	−1 9	−1 9	−2 0	−2 0	−2 0	−2 1	−2 1	−2 2	−2 3	−2 1	−2 0	−1 9	+ 10
+ 11	−2 0	−1 9	−2 1	−2 0	−2 0	−2 1	−2 2	−2 3	−2 4	−2 2	−2 2	−2 0	+ 11
+ 12	−2 1	−2 0	−2 1	−2 0	−2 1	−2 2	−2 3	−2 4	−2 4	−2 4	−2 3	−2 1	+ 12
+ 13	−2 2	−2 1	−2 1	−2 0	−2 1	−2 2	−2 3	−2 4	−2 5	−2 5	−2 4	−2 3	+ 13
+ 14	−2 3	−2 2	−2 1	−2 1	−2 2	−2 3	−2 4	−2 5	−2 6	−2 5	−2 5	−2 4	+ 14

By the application of the foregoing corrections to the errors of the moon's tabular right ascension, these errors may be supposed to be reduced to very small quantities, depending on the errors of the lunar elements, with which they are connected by the equation

$$\delta\alpha = \frac{d\alpha}{dl}\,\delta l + \frac{d\alpha}{d\theta}\,\delta\theta + \frac{d\alpha}{d\iota}\,\delta\iota,$$

the differential coefficients having the values given on page 12 When we substitute these values, the expression for $\delta\alpha$ will contain the terms

$$\begin{aligned}&(+\,018\,\delta\theta - 037\,\delta\alpha)\cos(2\,l - \theta)\\ &-\,087\,\delta\alpha\cos 2\,l\\ &+\,018\,\delta\theta\cos\theta\\ &+\,0\,21\,\delta\iota\sin\theta\\ &-\,0\,21\,\delta\iota\sin(2\,l-\theta)\end{aligned}$$

If we represent the sum of these terms by P, we shall have

$$\delta l = \delta\alpha - \mathrm{P}$$

In the investigation of the corrections to the moon's eccentricity and longitude of perigee, the terms of P may be entirely neglected This arises from the circumstances that the appreciable terms of l or α arising from the errors of these elements have the same period with g, the mean anomaly, while P contains no appreciable periodic term depending on g The outstanding portion of $\delta\alpha$ probably averages not more than one second or two at the utmost, so that the term 037 $\delta\alpha$ is quite insignificant The term .018 $\delta\theta$ may have a constant value of 0″ 25, more or less,* but the short period of the term $2\,l - \theta$, and its incommensurability with the period of g, permit of this error being regarded as fortuitous The same remark applies to the terms 087 $\delta\alpha$ cos $2\,l$ and 0 21 $\delta\iota$ sin $(2\,l - \theta)$ The only remaining terms have the period of θ, which is more than eighteen years The effect of these possible errors is therefore eliminated in the mean correction for each year, which has been already applied to the errors

To determine the correction to the eccentricity and longitude of the perigee resulting from each year's observations, the residuals in right ascension, after the application of the three corrections already described, have been arranged according to the values of the mean anomaly to which they correspond The results are shown in the following table, which gives for certain limits of mean anomaly in the first column, firstly, the sum of the residuals (tabular *minus* observed) in right ascension, corresponding to all the values of mean anomaly between those limits, and, secondly, the number of the residuals In taking these sums, the observations at the two observatories are counted separately, so that when observations were made at both observatories on the same date, the sum of the residuals is taken and the observations count 2 in the column N

* It is afterward found that the value of this product is only 0″ 08.

Sums of errors of moon's corrected right ascension, given by observations at Greenwich and Washington

Limits of mean anomaly	1862		1863		1864		1865	
	Σδα	N	Σδα	N	Σδα	N	Σδα	N
° °	″		″		″		″	
0 to 10	+ 3 9	4	+ 21 5	10	+ 19 6	9	+ 1 4	7
10 to 20	+ 3 6	6	+ 12 3	12	+ 6 1	7	+ 3 4	4
20 to 30	− 0 2	5	+ 14 2	8	+ 5 8	5	− 0 3	10
30 to 40	+ 9 3	8	+ 23 7	11	+ 4 5	7	− 0 5	5
40 to 50	+ 2 7	8	+ 9 0	8	+ 2 6	3	− 3 6	6
50 to 60	+ 0 3	8	+ 9 8	9	− 1 6	10	− 1 1	6
60 to 70	+ 8 9	10	− 4 3	7	+ 0 7	5	− 6 1	7
70 to 80	− 3 7	4	+ 7 0	10	− 7 0	6	− 6 7	6
80 to 90	+ 6 7	7	− 6 7	6	− 11 2	9	− 6 1	6
90 to 100	+ 3 9	6	− 3 3	9	− 3 4	6	− 8 5	7
100 to 110	+ 3 9	11	− 0 4	5	− 2 1	5	− 0 7	5
110 to 120	− 6 4	9	− 3 9	8	− 3 0	3	− 7 5	8
120 to 130	− 3 2	8	− 3 9	7	+ 0 1	5	− 5 5	6
130 to 140	− 7 8	6	− 8 8	8	− 12 2	7	+ 5 0	5
140 to 150	− 0 9	5	− 15 9	8	+ 0 9	3	+ 1 1	5
150 to 160	− 0 1	5	− 18 2	9	− 6 7	7	+ 1 5	4
160 to 170	− 8 8	4	− 19 7	6	+ 2 5	6	+ 4 3	5
170 to 180	− 5 7	4	− 9 9	7	− 5 3	5	+ 6 4	6
180 to 190	− 17 4	9	− 33 1	14	− 8 6	7	+ 8 9	6
190 to 200	− 15 5	7	− 4 3	4	− 0 6	4	+ 13 2	8
200 to 210	− 3 8	10	− 1 0	6	− 6 4	9	+ 7 8	8
210 to 220	− 0 2	2	− 1 9	9	− 2 9	8	+ 13 1	7
220 to 230	− 28 9	9	− 7 5	10	+ 3 6	7	+ 5 1	5
230 to 240	− 7 3	7	− 1 9	7	+ 0 8	7	+ 10 3	5
240 to 250	+ 13 0	8	+ 0 4	9	+ 1 6	7	+ 7 3	8
250 to 260	− 2 0	4	+ 7 6	8	+ 11 5	8	+ 7 3	7
260 to 270	+ 1 6	9	+ 1 4	5	+ 11 7	7	+ 16 2	12
270 to 280	+ 3 7	5	+ 11 3	9	+ 25 3	11	+ 7 6	11
280 to 290	+ 4 7	7	+ 0 8	5	+ 18 2	8	+ 9 6	8
290 to 300	− 1 3	1	+ 15 9	7	+ 6 6	4	+ 5 8	11
300 to 310	+ 3 0	3	+ 23 5	9	+ 7 8	6	+ 10 1	7
310 to 320	+ 2 3	2	+ 22 6	6	+ 6 4	5	+ 16 4	10
320 to 330	− 2 8	5	+ 18 2	9	+ 11 6	7	+ 14 5	7
330 to 340	+ 9 5	6	+ 1 2	7	+ 18 5	10	+ 16 7	11
340 to 350	+ 11 8	8	+ 7 2	7	+ 4 2	7	+ 7 6	7
350 to 360	+ 13 6	5	+ 14 4	8	+ 16 5	6	+ 5 3	9
	+106 4	225	+222 0	287	+187 1	236	+205 9	255
	−116 0		−144 7		− 71 0		− 46 6	
	− 9 6		+ 78 3		+116 1		+159 3	

Sums of errors of moon's corrected right ascension, &c—Continued.

Limits of mean anomaly	1866		1867		1868		1869	
	Σδα	N	Σδα	N	Σδα	N	Σδα	N
° °	″		″		″		″	
0 to 10	− 1 7	6	+ 7 4	5	− 4 2	4	− 10 7	4
10 to 20	− 2 5	4	− 5 0	2	+ 3 9	7	− 4 2	4
20 to 30	− 7 5	3	− 1 7	4	− 2 5	3	− 0 8	6
30 to 40	− 7 1	5	− 7 5	3	− 9 4	6	+ 4 2	5
40 to 50	− 14 5	7	+ 5 5	4	− 9 0	5	+ 11 0	6
50 to 60	− 0 7	1	− 2 0	4	− 0 7	7	+ 5 5	3
60 to 70	+ 1 3	5	− 8 5	4	+ 2 2	7	+ 3 1	5
70 to 80	+ 5 3	6	− 4 8	3	+ 4 1	8	+ 7 7	7
80 to 90	+ 1 6	6	− 3 6	1	+ 12 2	7	+ 8 0	8
90 to 100	+ 3 9	4	+ 2 6	5	− 0 3	4	+ 16 8	8
100 to 110	+ 4 4	9	− 0 6	5	+ 14 9	7	+ 5 1	9
110 to 120	+ 4 2	8	+ 3 9	5	+ 9 8	6	+ 8 3	6
120 to 130	− 5 4	8	+ 1 6	7	+ 4 1	5	+ 14 5	7
130 to 140	+ 3 4	6	+ 4 1	6	+ 10 2	8	+ 7 5	8
140 to 150	+ 10 1	9	+ 1 9	7	+ 5 2	7	+ 3 1	6
150 to 160	− 4 1	6	− 2 6	7	+ 2 1	9	+ 20 3	7
160 to 170	+ 3 3	7	+ 6 8	5	+ 1 3	8	+ 3 7	3
170 to 180	− 0 1	7	− 5 0	8	+ 0 8	7	+ 12 2	7
180 to 190	+ 0 8	6	− 0 3	2	+ 12 3	8	+ 7 0	5
190 to 200	+ 5 9	6	+ 2 0	4	+ 17 9	6	+ 6 3	4
200 to 210	− 3 2	6	+ 2 8	6	+ 5 2	5	+ 10 1	5
210 to 220	+ 0 3	6	− 1 7	4	+ 13 0	8	+ 12 2	5
220 to 230	− 5 4	4	+ 12 9	9	+ 4 8	4	+ 12 3	7
230 to 240	+ 4 1	8	+ 8 2	6	+ 15 2	9	− 1 3	3
240 to 250	− 1 8	7	+ 25 4	9	+ 7 4	8	− 6 4	6
250 to 260	+ 9 4	7	+ 0 9	3	+ 14 2	8	− 3 6	2
260 to 270	+ 2 7	7	+ 11 7	6	− 5 0	2	− 17 3	7
270 to 280	+ 9 7	4	+ 3 3	4	+ 1 0	7	− 18 8	5
280 to 290	+ 11 6	12	+ 7 0	7	− 9 1	5	− 21 4	6
290 to 300	+ 4 0	4	+ 0 7	3	− 3 2	8	− 13 6	3
300 to 310	+ 6 7	4	+ 16 5	7	− 8 0	2	− 4 8	2
310 to 320	+ 3 4	2	+ 2 3	5	− 13 8	8	− 0 8	1
320 to 330	+ 7 7	5	+ 0 2	5	− 10 6	9	− 4 2	2
330 to 340	+ 9 1	5	+ 3 5	6	− 11 7	6	− 18 5	6
340 to 350	+ 10 8	6	− 5 4	7	− 9 8	5	− 10 6	4
350 to 360	+ 9 2	7	− 7 2	4	− 18 3	6	− 2 2	5
	+132 9	213	+131 2	182	+161 8	229	+178 9	187
	− 54 0		− 55 9		−115 6		−139 2	
	+ 78 9		+ 75 3		+ 46 2		+ 39 7	

Sums of errors of moon's corrected right ascension, &c—Concluded

Limits of mean anomaly	1870		1871		1872		1873		1874	
	Σδα	N	Σδα	N	Σδα	N	Σδα	N	Σδα	N
° °	″		″		″		″		″	
0 to 10	− 7 2	5	− 3 2	5	+ 6 5	6	− 4 3	6	+ 4 6	4
10 to 20	− 2 2	5	+ 1 7	11	+ 8 5	10	+ 5 2	4	+ 5 9	5
20 to 30	+ 5 1	6	− 0 3	7	+ 5 5	8	+ 5 2	8	+ 12 5	6
30 to 40	+ 10 7	8	+ 6 4	7	+ 11 8	7	+ 3 4	3	+ 5 1	5
40 to 50	+ 11 2	8	+ 16 7	9	+ 6 0	4	+ 6 6	4	+ 4 4	5
50 to 60	− 7 1	5	+ 9 7	6	+ 13 2	6	+ 4 1	7	+ 2 1	5
60 to 70	+ 1 0	9	+ 18 9	8	+ 10 4	3	+ 13 4	6	+ 10 1	4
70 to 80	− 2 6	5	+ 10 2	7	+ 12 4	8	+ 13 5	3	+ 6 6	6
80 to 90	+ 12 0	12	+ 11 7	5	+ 11 3	4	+ 15 8	7	+ 6 0	3
90 to 100	+ 10 1	8	+ 12 5	3	+ 9 8	4	+ 5 1	2	+ 5 9	7
100 to 110	+ 10 8	4	+ 19 7	8	+ 13 0	6	+ 1 5	3	+ 10 9	6
110 to 120	+ 5 8	6	+ 8 2	4	+ 18 7	6	+ 5 3	2	+ 4 6	4
120 to 130	+ 10 1	7	+ 9 7	5	+ 18 3	7	+ 6 1	5	− 4 7	6
130 to 140	+ 10 1	5	+ 15 4	5	+ 0 2	2	+ 3 3	3	+ 1 8	3
140 to 150	+ 18 2	8	+ 2 1	3	+ 2 9	3	+ 8 4	5	− 0 8	7
150 to 160	+ 4 4	3	+ 3 0	7	+ 2 1	8	− 3 9	3	+ 1 3	5
160 to 170	+ 8 8	5	+ 8 7	4	+ 6 6	5	− 5 4	4	− 10 1	9
170 to 180	+ 6 9	3	+ 6 2	6	− 1 2	3	− 1 7	3	− 1 0	6
180 to 190	+ 1 8	1	+ 3 9	4	+ 1 9	4	− 2 2	4	+ 5 0	6
190 to 200	+ 7 5	4	+ 3 5	3	− 1 2	5	− 6 6	6	− 1 0	2
200 to 210	+ 2 1	5	+ 1 0	3	− 2 2	6	− 0 9	2	+ 3 7	6
210 to 220	− 2 3	2	− 2 6	2	+ 1 2	3	− 6 6	3	− 5 0	5
220 to 230	− 2 5	3	− 9 3	7	− 7 2	5	− 0 1	4	− 16 0	7
230 to 240	− 0 4	5	− 3 2	6	− 4 8	5	− 3 5	1	− 13 5	4
240 to 250	− 9 7	5	− 9 1	8	− 6 5	3	− 7 5	5	− 15 1	8
250 to 260	− 12 1	6	− 5 2	5	− 9 1	4	− 7 1	4	− 23 0	5
260 to 270	− 2 3	2	− 4 6	5	− 13 8	8	− 8 6	3	− 22 6	4
270 to 280	− 12 9	8	− 7 1	7	− 8 4	5	− 4 3	4	− 15 6	4
280 to 290	− 5 6	3	− 2 7	6	− 16 7	9	− 10 8	6	− 9 1	3
290 to 300	− 5 5	4	+ 4 0	4	− 10 3	8	− 9 8	4	− 13 4	8
300 to 310	− 4 0	4	− 9 5	6	− 9 5	5	− 1 8	1	− 9 1	9
310 to 320	− 8 7	3	− 6 6	5	− 5 6	4	− 3 2	4	− 5 3	6
320 to 330	− 13 5	6	− 4 9	7	− 8 5	5	− 11 3	7	− 1 4	7
330 to 340	− 9 7	4	− 2 8	7	− 8 5	5	− 0 3	8	− 4 3	3
340 to 350	− 3 6	3	− 1 7	4	− 5 1	5	− 9 2	6	+ 2 2	11
350 to 360	− 8 7	5	+ 6 3	4	+ 0 1	6	− 4 0	5	+ 2 5	6
	+136 6	185	+179 5	203	+160 4	195	+ 96 9	155	+ 95 2	200
	−120 6		− 72 8		−118 6		−113 1		−171 0	
	+ 16 0		+106 7		+ 41 8		− 16 2		− 75 8	

Neglecting all terms multiplied by the eccentricity in the coefficients, each residual gives an equation of the form

$$\Delta l + 2 \sin g \, \Delta e - 2 \cos g \, e \, \Delta \pi = r$$

or, putting

$$h = 2\,\Delta\delta e = -2\,\delta e$$
$$k = -2\,\Delta e\,\delta\pi = 2\,e\,\delta\pi$$

the equation will be

$$\Delta l + h \sin g + k \cos g = r,$$

Δe and $\Delta\pi$ being the *errors* of the tabular eccentricity and longitude of the perigee, while δe and $\delta\pi$ represent the corresponding *corrections*

The equations are now solved as if all the residuals within each pair of 20° limits corresponded to the mean of the limit,—that is, as if all between 0° and 20° corresponded to $g = 10°$, those between $g = 20°$ and $g = 40°$ to $g = 30°$, and so on If, then, we put

$g_1 = 10°$, $g_2 = 30°$, etc ,

r_i, the sum of all the residuals in any one year corresponding to $g = g_i$;

n_i, the corresponding number of observations,

$s_i = \sin g_i$,

$c_i = \cos g_i$

the normal equations for determining δl, h, and k, by least squares, will be

$$(\Sigma n_i)\ \Delta l + (\Sigma n_i s_i)\ h + (\Sigma n_i c_i)\ k = \Sigma r_i$$
$$(\Sigma n_i s_i)\ \Delta l + (\Sigma n_i s_i^2)\ h + (\Sigma n_i s_i c_i)\ k = \Sigma s_i r_i$$
$$(\Sigma n_i c_i)\ \Delta l + (\Sigma n_i s_i c_i)\ h + (\Sigma n_i c_i^2)\ k = \Sigma c_i r_i$$

The formation and solution of these equations for each year give the following values of the outstanding errors of the lunar elements for each year

	$''$	$''$
1862,	$h = +0\ 04$	$k = +1\ 23$
1863,	$-0\ 64$	$+1\ 78$
1864,	$-1\ 07$	$+1\ 09$
1865,	$-1\ 03$	$-0\ 15$
1866,	$-0\ 47$	$+0\ 10$
1867,	$-0\ 93$	$-0\ 36$
1868,	$+0\ 34$	$-1\ 46$
1869,	$+1\ 67$	$-1\ 56$
1870,	$+1\ 48$	$-1\ 14$
1871,	$+1\ 65$	$-0\ 36$
1872,	$+2\ 15$	$-0\ 12$
1873,	$+1\ 91$	$+0\ 16$
1874,	$+1\ 92$	$+0\ 60$

The periodic character of these residuals is very remarkable, indicating, as it does, either a hitherto unknown inequality of the moon's mean longitude, having nearly the same period with the orbital revolution, or one of the eccentricity and longitude of perigee, having a period of between fifteen and twenty years To investigate this inequality, we shall assume that each value of h is of the form

$$h - \alpha \sin(\mu + nt)$$

and each value of k of the form

$$k + \alpha' \cos(\mu' + n't),$$

h, k, α, α', μ, μ', n, and n' being unknown quantities to be determined, and t the time in years from any assumed epoch We shall take for the epoch the middle of the period through which the observations extend, that is, 1868 5 If, then, we represent the thirteen values of h and k in chronological order by $h_{-6}, h_{-5}, \ldots, h_6, k_{-6}, k_{-5}, \ldots, k_6$, the equations of condition for h and k respectively may be put into the form

$$h_i = h - \alpha \sin \mu \cos i n - \alpha \cos \mu \sin i n$$

$$k_i = k + \alpha' \cos \mu \cos i n - \alpha' \sin \mu \sin i n$$

Regarding h, k, $\alpha \sin \mu$, $\alpha \cos \mu$, α' $\sin \mu$, and α' $\cos \mu$ as the unknown quantities, the normal equations for determining these quantities are

(1) *From the values of* h_i

$$13\, h - (\Sigma \cos i n)\, \alpha \sin \mu = \Sigma h_i$$

$$-(\Sigma \cos i n)\, h + (\Sigma \cos^2 i n)\, \alpha \sin \mu = -\Sigma h_i \cos i n$$

$$(\Sigma \sin^2 i n)\, \alpha \cos \mu = -\Sigma h_i \sin i n$$

(2) *From the values of* k_i

$$13\, k + (\Sigma \cos i n)\, \alpha' \cos \mu' = \Sigma k_i$$

$$(\Sigma \cos i n)\, k + (\Sigma \cos^2 i n)\, \alpha' \cos \mu' = \Sigma k_i \cos i n$$

$$\Sigma\, (\sin^2 i n)\, \alpha' \sin \mu' = -\Sigma k_i \sin i n$$

It will be observed that all the coefficients having as a factor either $\Sigma \sin i n$ or $\Sigma \sin i n \cos i n$ vanish

The value of n apparently is not readily determined directly by least squares we shall therefore assume several values of this quantity, and ascertain by which value the conditions can best be satisfied The following are the abbreviated values of the purely trigonometric summations

$$\Sigma \cos i n = \frac{\sin 6\tfrac{1}{2} n}{\sin \tfrac{1}{2} n} = c$$

$$\Sigma \cos^2 i n = \frac{13 \sin n + \sin 13 n}{2 \sin n} = c_1$$

$$\Sigma \sin^2 i n = \frac{13 \sin n - \sin 13 n}{2 \sin n} = s_1$$

If we solve the preceding equations, and put, for brevity,

$$C = \frac{c}{13\, c_1 - c^2}$$

$$C_1 = \frac{c_1}{13\, c_1 - c^2}$$

$$C_2 = \frac{13}{13\, c_1 - c^2}$$

the resulting expressions for the unknown quantities are

$$h = C_1 \Sigma h_i - C \Sigma h_i \cos i n$$

$$\alpha \sin \mu = C \Sigma h_i - C_2 \Sigma h_i \cos i n$$

$$\alpha \cos \mu = -\frac{1}{s_1} \Sigma h_i \sin i n$$

$$k = C_1 \Sigma k_i - C \Sigma k_i \cos i n$$

$$\alpha' \cos \mu' = - C \Sigma k_i + C_2 \Sigma k_i \cos i n$$

$$\alpha' \sin \mu' = -\frac{1}{s_1} \Sigma k_i \sin i n$$

The period of h and k lies probably between fifteen and twenty years, which would make the value of n, or the annual motion of the inequality, lie between 18° and 24° The following are the values of the various quantities depending on n for the different values of n between these limits

n	log c	log c_1	log s_1	log C	log C	log C_2
°						
18	0 756	0 715	0 893	9 213	9 172	9 571
19	0 705	0 707	0 898	9 097	9 099	9 506
20	0 644	0 705	0 900	8 977	9 038	9 447
21	0 577	0 709	0 897	8 858	8 990	9 395
22	0 498	0 718	0 891	8 734	8 954	9 350
23	0 406	0 731	0 882	8 604	8 929	9 312
24	0 291	0 747	0 870	8 453	8 909	9 276
25	0 143	0 765	0 856	8 275	8 897	9 246

n	$\Sigma h_i \sin in$	$\Sigma h_i \cos in$	$\Sigma k_i \sin in$	$\Sigma k_i \cos in$
°	''	''	''	''
18	+ 11 48	+ 1 96	− 4 66	− 4 66
19	+ 11 66	+ 1 52	− 4 68	− 5 04
20	+ 11 78	+ 1 09	− 4 69	− 5 40
21	+ 11 83	+ 0 68	− 4 68	− 5 73
22	+ 11 81	+ 0 29	− 4 66	− 6 04
23	+ 11 73	− 0 08	− 4 62	− 6 33
24	+ 11 58	− 0 44	− 4 57	− 6 60
25	+ 11 37	− 0 78	− 4 50	− 6 86

The preceding equations now give the following separate values of the unknown quantities, corresponding to the various assumed values of n ·

n	h	a	μ	k	a'	μ'
°	''	''	°	''	''	°
18	0 72	1 53	164 0	0 73	1 81	160 8
19	0 69	1 53	165 2	0 61	1 71	159 7
20	0 66	1 53	166 3	0 49	1 62	158 5
21	0 63	1 54	167 2	0 39	1 53	157 2
22	0 61	1 55	168 1	0 31	1 47	156 0
23	0 60	1 57	169 0	0 23	1 42	154 8
24	0 58	1 59	169 8	0 17	1 39	153 6
25	0 56	1 61	170 4	0 11	1 36	152 6

There can be little serious doubt that in the case of the present inequality the theoretical values of μ and μ' should be the same, and it is also probable that those of α and α' may be substantially identical. The small differences between the values of α and α' and of μ and μ' add so much weight to this probability that we shall make

another solution of the equations on the supposition that $\alpha' = \alpha$ and $\mu' = \mu$ The normal equations then become

$$\begin{aligned}
13\,h - c\alpha \sin\mu &= \Sigma h_\iota \\
-c\,h + 13\,\alpha \sin\mu &= -\Sigma h_\iota \cos \iota n - \Sigma k_\iota \sin \iota n = S_1 \\
13\,k + c\alpha \cos\mu &= \Sigma k_\iota \\
c\,k + 13\,\alpha \cos\mu &= \Sigma k_\iota \cos \iota n - \Sigma h_\iota \sin \iota n = S_2
\end{aligned}$$

The solution of these equations is

$$h = \frac{13}{13^2 - c^2}\,\Sigma h_\iota + \frac{c}{13^2 - c^2}\,S_1$$

$$k = \frac{13}{13^2 - c^2}\,\Sigma k - \frac{c}{13^2 - c^2}\,S_2$$

$$\alpha \sin\mu = \frac{13}{13^2 - c^2}\,S_1 + \frac{c}{13^2 - c^2}\,\Sigma h_\iota$$

$$\alpha \cos\mu = \frac{13}{13^2 - c^2}\,S_2 - \frac{c}{13^2 - c^2}\,\Sigma k_\iota$$

A comparison of the separate solutions of the equations in h and k shows that the value of n which best satisfies the conditions lies between 22° and 25° The values of h, k, α, and μ were therefore derived only from the last equations for the last four values of n For each of these separate values of n, the corresponding values of h_ι and k_ι were computed from the formulæ

$$\begin{aligned}
h_\iota &= h - \alpha \sin(\mu + \iota n) \\
k_\iota &= k + \alpha \cos(\mu + \iota n)
\end{aligned}$$

in which, it will be remembered, the index ι is simply the number of the year from 1868, so that we have,

For 1862, $\iota = -6$
For 1863, $\iota = -5$
etc, etc

These computed values of h_ι and k_ι were then compared with the values derived directly from observations, and given on page 20, and the sum of the squares of the outstanding residuals was taken The values of the unknown quantities, together with the sum of the squares of the residuals, are as follow

n	h	k	a	μ	Σ
°	″	″	″	°	″
22	+ 0 66	+ 0 34	1 54	161 2	3 207
23	+ 0 63	+ 0 27	1 52	161 3	3 170
24	+ 0 61	+ 0 20	1 51	161 5	3 246
25	+ 0 58	+ 0 14	1 49	161 7	3 441

The sum of the squares becomes a minimum for $n = 22°\ 8$, showing a period of the inequality of 15ʸ 8, with a possible error of a year or more The formulæ for h_ι and k_ι thus become

$$\begin{aligned}
h_\iota &= +0''\,64 - 1''\,52 \sin(161°\,2 + 22°\,8\,\iota) \\
k_\iota &= +0''\,28 + 1''\,52 \cos(161°\,2 + 22°\,8\,\iota)
\end{aligned}$$

from which we have the following comparison of the computed and observed values of h_i and k_i

Year	h_i			k_i		
	C	O	O − C	C	O	O − C
	″	″	″	″	″	″
1862	+ 0 01	+ 0 04	+ 0 03	+ 1 67	+ 1 23	− 0 44
1863	− 0 48	− 0 64	0 16	+ 1 32	+ 1 78	+ 0 46
1864	− 0 79	− 1 07	− 0 28	+ 0 80	+ 1 09	+ 0 29
1865	− 0 88	− 1 03	− 0 15	+ 0 22	− 0 15	− 0 37
1866	− 0 74	− 0 47	+ 0 27	− 0 38	+ 0 10	+ 0 48
1867	− 0 37	− 0 93	− 0 56	− 0 85	− 0 36	+ 0 49
1868	+ 0 14	+ 0 34	+ 0 20	− 1 16	− 1 46	− 0 30
1869	+ 0 74	+ 1 67	+ 0 93	− 1 23	− 1 56	− 0 33
1870	+ 1 33	+ 1 48	+ 0 15	− 1 07	− 1 14	− 0 07
1871	+ 1 80	+ 1 65	− 0 15	− 0 70	− 0 36	+ 0 34
1872	+ 2 09	+ 2 15	+ 0 06	− 0 18	− 0 12	+ 0 06
1873	+ 2 15	+ 1 91	− 0 24	+ 0 42	+ 0 16	− 0 26
1874	+ 1 98	+ 1 92	− 0 06	+ 1 00	+ 0 60	− 0 40

The probable residual for each year is 0″ 27

We have supposed the hypothetical inequality of longitude to be of the form

$$\Delta v = h_i \sin g + k_i \cos g$$

Substituting in this the periodic part of h_i and k_i, and replacing i by t, which now represents the time in years from 1868 5, it becomes

$$\Delta v = 1'' \, 52 \sin (g + 251^\circ \, 2 + 22^\circ \, 8 \, t)$$

or

$$\Delta v = 1'' \, 52 \sin [g + 22^\circ \, 8 \, (Y - 1857 \, 5)]$$

The entirely unexpected character of the periodic term thus brought to light renders its verification by a longer series of observations very desirable For this purpose, we need comparisons of observations previous to 1862 with Hansen's tables, because none of the older tables with which comparisons have been made are accurate enough for the purpose Now, the Greenwich Observations for 1859 contain, as an appendix, a comparison of the longitudes and latitudes from Hansen's tables with Greenwich observations from 1847 to 1858 inclusive, and I have utilized the comparison of the longitudes derived from meridian observations in the following way

A list of limiting dates to tenths of a day was made out, including the whole twelve years, and showing between what dates the moon's mean anomaly was found in each sextant The sum of the errors in longitude given by the meridian observations was then taken during the period that the anomaly was found in each sextant None of the corrections found in the first part of this discussion were applied, for the reason that most of them could be treated as accidental errors, and the means could be taken so as nearly to eliminate the effects of the larger ones A specimen of the form chosen is here given Under each of the several values of g, given at the tops of the several

columns, is shown, firstly, the date at which g had that particular value, and, secondly, the sum of the residuals in longitude during the period of $4^{d}\,6$ between that date and the one next following, together with the number of the residuals, the latter being in small subscript figures

$g = 0 +$			$g = 60^\circ +$			$g = 120^\circ +$			$g = 180^\circ +$			$g = 240^\circ +$			$g = 300^\circ +$		
	1847	″		1847	″		1847	″		1847	″		1847	″		1847	″
Jan	19 0	− 2 9_{1}	Jan	24 2	− 3 1	Jan	1 2	+ 1 3	Jan	5 8		Jan	10 4	+ 2 9_{1}	Jan	15 0	
Feb	16 1	− 1 0_{1}	Feb	20 7	+ 0 4_{1}	Jan	28 8	+ 3 1_{2}	Feb	2 4	+ 3 7_{1}	Feb	7 0	+ 5 6_{3}	Feb	11 6	
Mar	15 7		Mar	20 3	− 3 0_{1}	Feb	25 3	+ 4 5_{1}	Mar	1 9	+ 3 7_{1}	Mar	6 5	+ 2 3_{1}	Mar	11 1	
April	12 3		April	16 9		Mar	24 9	+ 6 1_{1}	Mar	29 5	− 0 4_{1}	April	3 1		April	7 7	+ 3 2_{1}
May	9 8		May	14 4		April	21 5	+ 3 2_{1}	April	26 0	+ 2 7_{1}	April	30 6		May	5 2	+ 1 0_{1}
June	6 2	+ 2 8_{1}	June	10 8		May	19 0	+ 2 8_{2}	May	23 6	+ 3 9_{1}	May	28 2	− 0 3_{6}	June	1 8	+ 4 1_{1}
July	3 8	+ 4 4_{0}	July	8 4		June	15 4	− 1 4_{1}	June	20 0	− 0 6_{2}	June	24 6	− 2 1_{1}	June	29 2	+ 1 9_{1}
Aug	0 4	− 0 3_{1}	Aug	5 0		July	13 0		July	17 6	− 1 6_{1}	July	22 2	+ 2 9_{1}	July	26 8	+ 6 8_{1}
Aug	28 0	+ 5 9_{1}	Sept	1 6	+ 3 8_{1}	Aug	9 6		Aug	14 2		Aug	18 8	− 8 4_{1}	Aug	23 4	+ 8 1_{2}
Sept	24 6	+ 12 2_{1}	Sept	29 2		Sept	6 2		Sept	10 8		Sept	15 4	+ 3 6_{1}	Sept	20 0	+ 1 0_{1}
Oct	22 1	+ 12 2_{1}	Oct	26 7	+ 8 7_{2}	Oct	3 8		Oct	8 4		Oct	13 0		Oct	17 6	− 7 3_{1}
Nov	19 7	− 1 2_{1}	Nov	23 3	+ 9 3_{1}	Oct	31 3	+ 1 0_{1}	Nov	4 9		Nov	9 5		Nov	14 1	0 0_{1}
Dec	16 1	− 3 4_{2}	Dec	20 7		Nov	27 6	+ 11 4_{1}	Dec	2 2		Dec	6 8		Dec	11 4	− 1 6_{2}
						Dec	25 3	+ 0 7_{1}	Dec	29 9	+ 0 7_{1}	Dec	34 5		Dec	39 1	− 2 8_{1}
	1848			1848			1848			1848			1848			1848	
Jan	12 7	− 7 3	Jan	17 3		Jan	21 9		Jan	26 5	+ 0 2_{1}	Jan	31 1		Feb	4 7	
Feb	9 3	− 8 1_{1}	Feb	13 9	− 6 2_{6}	Feb	18 5	− 1 4_{2}	Feb	23 1	− 1 1_{1}	Feb	27 7	− 0 8_{1}	Mar	3 3	
Mar	7 9	− 1 8_{1}	Mar	12 5	− 4 3_{1}	Mar	17 1	+ 4 7_{1}	Mar	21 7		Mar	26 3		Mar	30 9	
April	4 5		April	9 1	− 4 1_{1}	April	13 7	− 1 5_{1}	April	18 3	− 1 8_{1}	April	22 9	+ 2 9_{1}	April	27 5	
May	2 0		May	6 6	− 7 1_{0}	May	11 2	+ 1 0_{1}	May	15 8	+ 1 4_{1}	May	20 4	+ 9 0_{1}	May	25 0	
May	29 6		June	3 2	− 8 9_{1}	June	7 8	− 0 9_{1}	June	12 4	− 0 2_{2}	June	17 0		June	21 6	+ 2 1_{1}
June	26 2		June	30 8	− 2 6_{1}	July	5 4	− 0 6_{1}	July	10 0	− 4 8_{1}	July	14 6	+ 10 4_{1}	July	19 2	− 0 1_{2}
July	23 7		July	28 3		Aug	1 9	− 5 1	Aug	6 5	− 3 1_{1}	Aug	11 1		Aug	15 7	+ 17 4_{1}
Aug	20 3	+ 1 2_{1}	Aug	24 9		Aug	29 5		Sept	3 1	− 6 7_{2}	Sept	7 7	− 5 1_{1}	Sept	12 3	+ 15 7_{1}
Sept	16 9	+ 22 5_{1}	Sept	21 5		Sept	26 1		Sept	30 7	− 1 0	Oct	5 3	+ 2 0_{1}	Oct	9 8	+ 8 3_{4}
Oct	14 1	+ 5 1_{1}	Oct	19 0		Oct	23 6		Oct	28 2		Nov	1 8	+ 1 9	Nov	6 4	− 4 9_{1}
Nov	11 0	+ 6 9_{1}	Nov	15 6	+ 12 8_{1}	Nov	20 2		Nov	24 8		Nov	29 4	− 9 4_{1}	Dec	4 0	− 7 2_{1}
Dec	8 5	− 6 5_{1}	Dec	13 1	+ 5 1_{0}	Dec	17 7	+ 7 0_{2}	Dec	22 3		Dec	26 9		Dec	31 5	− 5 7_{1}

If we follow any one of these vertical columns, we shall find that the dates correspond successively to all points of the lunation in a period of 412 days The first observations of each period will be the last ones of the lunation, and the last ones those made immediately after new moon Between each pair of periods will be a gap, generally of three or four months, during which the moon was, at the corresponding points of mean anomaly, too near the sun to be observed If the observations are equally scattered through each period, all the errors arising from erroneous semi-diameter and parallactic inequality will be eliminated The general minuteness of these errors, and their approach to a balance during each of the periods in question, are such as to render them insignificant, if we take the mean results, not by years, but by periods This is the course adopted; the partial periods at the beginning and end of the entire series of observations being omitted The first period actually employed was that corresponding

4 M

to the sextant 240°–300°, in which the first observation was made on January 10, 1847, and the last on September 18 of the same year. The last period corresponded to the sextant 180°–240°, the last observation in which was on November 13, 1858. There were, in all, ten periods corresponding to each sextant, and hence ten sets of equations, each giving mean values of h, k, and δl for periods extending through a little more than a year. Each residual gave an equation of condition, for the coefficients of which the mean value corresponding to the entire sextant was taken. These values for the several sextants are as follow:

i	g	$\sin g$	$\cos g$	$\sin^2 g$	$\sin g \cos g$	$\cos^2 g$
	° °					
1	0 – 60	+ 0 48	+ 0 83	0 23	+ 0 40	0 69
2	60 – 120	+ 0 96	0 00	0 91	0 00	0 00
3	120 – 180	+ 0 48	− 0 83	0 23	− 0 40	0 69
4	180 – 240	− 0 48	− 0 83	0 23	+ 0 40	0 69
5	240 – 300	− 0 96	0 00	0 91	0 00	0 00
6	300 – 360	− 0 48	+ 0 83	0 23	− 0 40	0 69

The sums of the residual errors, corresponding to each period and each sextant arranged in chronological order, together with the number of residuals of which each sum is formed, are as follow:

Mean date	$i=5$	$i=6$	$i=1$	$i=2$	$i=3$	$i=4$
	″	″	″	″	″	″
1847 8	+ 6 5	+ 14 4$_{20}$	+ 15 4$_{22}$	− 11 7$_{22}$	+ 9 0$_{21}$	− 16 7$_{17}$
1848 9	+ 6 7	+ 8 7	− 33 0$_{27}$	− 1 9$_{22}$	+ 23 2$_{18}$	+ 31 5$_{17}$
1850 1	+ 1 5	− 34 1$_{17}$	− 40 9$_{20}$	− 9 1$_{26}$	+ 22 2$_{22}$	+ 33 9$_{25}$
1851 2	− 4 5	− 59 4$_{19}$	− 50 7$_{19}$	− 23 5$_{21}$	− 4 8$_{21}$	+ 20 6
1852 4	− 42 8	− 50 0$_{22}$	− 48 0$_{17}$	− 21 5$_{18}$	+ 35 0$_{20}$	+ 25 4
1853 5	− 31 2	−106 9	− 63 6	+ 1 2$_{17}$	+ 6 0$_{21}$	− 38 0$_{2}$
1854 6	− 30 3$_{17}$	− 94 6$_{21}$	− 35 4$_{27}$	+ 4 2$_{28}$	+ 1 7$_{11}$	− 24 4$_{20}$
1855 8	− 24 3$_{14}$	− 30 0$_{16}$	− 7 3$_{20}$	− 6 9$_{19}$	− 22 8$_{19}$	− 41 0$_{19}$
1856 9	− 36 2	− 23 8$_{11}$	+ 15 4$_{11}$	+ 4 2$_{26}$	− 48 5$_{21}$	− 77 0$_{17}$
1858 1	− 54 9	− 48 9$_{2}$	− 56 7$_{31}$	− 47 6$_{19}$	− 76 9$_{23}$	− 46 2$_{18}$

The dates given in the left-hand column are those corresponding to the mean of each horizontal line.

Putting s_i for the mean value of $\sin g$ corresponding to the index i, as already given, c_i for that of $\cos g$, and n_i for the corresponding number of observations, the normal equations are

$$
\begin{aligned}
n_i \Delta l + (\Sigma n_i s_i)\, h + (\Sigma n_i c_i)\, k &= \Sigma r_i \\
(\Sigma n_i s_i) \Delta l + (\Sigma n_i s_i^2)\, h + (\Sigma n_i s_i c_i)\, k &= \Sigma s_i r_i \\
(\Sigma n_i c_i) \Delta l + (\Sigma n_i s_i c_i)\, h + (\Sigma n_i c_i^2)\, k &= \Sigma c_i r_i
\end{aligned}
$$

The values of h and k thus given by the normal equations formed from the system of residuals shown in each horizontal line are shown in the next table, which also shows

the way in which they are treated For the sake of completeness, the corresponding quantities already found for the period 1862–74 are added, and included in the discussion, which now proceeds as follows, the method adopted being one which, though less rigorous than the former one, will show in a stronger light the evidence on which the new inequality depends

As the basis of the discussion, we take the independent values of h and k, derived from each series of observations, which values are given in the second and third columns of the table A preliminary comparison of the first series of values (1847–58) with the values of h and k derived from the formulæ already given indicates a diminution of the constant terms of those quantities, so that, instead of $+0''\,64$ and $+0''\,28$, they become, as a first approximation,

$$h_0 = +0''\,50$$
$$k_0 = +0''\,10$$

These constants are now subtracted from the values of h and k, leaving a series of residuals given in the fourth and fifth columns, which, if the periodic term under investigation has no existence, should be regarded as due to errors of observation, and, in the contrary case, should be representable by the formulæ

$$h' = -\alpha \sin(\mu + nt) + \text{accidental errors}$$
$$k' = \alpha \cos(\mu + nt) + \text{accidental errors}$$

To show clearly how far they are thus represented, we determine a coefficient, α, and an angle, N, by the equations

$$\alpha \sin N = -h'$$
$$\alpha \cos N = k'$$

The next two columns give the several values of α and N thus obtained The nearly regular progression of the angle N is too striking to be overlooked To see how nearly this angle can be represented as one increasing uniformly with the time, we solve the necessary equations of condition by least squares It is obvious that the greater the value of α the more certain will be the value of N we therefore give weights proportional to α Moreover, weights nearly twice as great in proportion are given to the second series (1862–74) as containing the results from two observatories, and being more carefully corrected The values of μ and n thus obtained by the method of least squares are

$$\mu = 164^\circ\,6 \pm 4^\circ\,4$$
$$n = 20\;\;8 \pm 0\;\;47$$

The probable error of a value of N of weight unity comes out

$$\pm 33^\circ$$

The residuals still outstanding are shown in the column ΔN This value of n is 2° less than that found from the second series of observations alone, and an examination of the residuals shows that there is a real discordance between the values of the angular motion of N given by the two series It is quite likely that the relative weights assigned

to the older series of observations are twice as great as they should be, and that the most probable value of the angle N lies nearly half-way between the two values

$$161^\circ 2 + 22^\circ 8 \, (t - 1868 \; 5)$$

and

$$164^\circ 6 + 20^\circ 8 \, (t - 1868 \; 5)$$

found from the last series alone, and from the two combined. I judge that the most probable value is

$$N = 163^\circ 2 + 21^\circ 6 \, (t - 1868 \; 5),$$

and that the probable error of the annual motion is more than half a degree, but less than a degree. The column $\Delta' N$ shows the residuals given by this value of N

Mean date	h	k	h	k'	a	N	Wt	$\mu + nt$	ΔN	$\Delta' N$
	"	"	"	"	"	°		°	°	
1847 8	− 0 08	+ 0 55	− 0 58	+ 0 45	0 74	52	1	94	+ 42	+ 24
1848 9	− 0 55	− 1 38	− 1 05	− 1 48	1 82	145	3	118	− 27	− 44
1850 1	− 0 20	− 1 91	− 0 70	− 2 01	2 13	161	3	141	− 20	− 36
1851 2	− 0 32	− 1 92	− 0 82	− 2 02	2 18	158	3	105	+ 7	− 8
1852 4	+ 0 26	− 2 45	− 0 24	− 2 55	2 56	175	4	189	+ 14	0
1853 5	+ 1 10	− 1 88	+ 0 60	− 1 98	2 07	197	3	212	+ 15	+ 2
1854 6	+ 1 45	− 1 40	+ 0 95	− 1 50	1 77	212	3	236	+ 24	+ 12
1855 8	+ 0 77	+ 0 31	+ 0 27	+ 0 21	0 34	308	4	260	− 48	− 60
1856 9	+ 1 76	+ 1 82	+ 1 26	+ 1 72	2 13	328	3	284	− 44	− 55
1858 1	− 0 17	+ 0 66	− 0 67	+ 0 56	0 88	50	1	307	−103	−112
1862 5	+ 0 04	+ 1 23	− 0 46	+ 1 13	1 22	22	3	40	+ 18	+ 12
1863 5	− 0 64	+ 1 78	− 1 14	+ 1 68	2 03	34	5	61	+ 27	+ 21
1864 5	− 1 07	+ 1 09	− 1 57	+ 0 99	1 85	58	5	81	+ 23	+ 19
1865 5	− 1 03	− 0 15	− 1 53	− 0 25	1 55	99	4	102	+ 3	− 1
1866 5	− 0 47	+ 0 10	− 0 97	0 00	0 97	90	2	123	+ 33	+ 30
1867 5	− 0 93	− 0 36	− 1 43	− 0 46	1 50	108	4	144	+ 36	+ 34
1868 5	+ 0 34	− 1 46	− 0 16	− 1 56	1 57	174	4	165	− 9	− 11
1869 5	+ 1 67	− 1 56	+ 1 17	− 1 66	2 03	215	5	185	− 30	− 30
1870 5	+ 1 48	− 1 14	+ 0 98	− 1 24	1 58	218	5	206	− 12	− 12
1871 5	+ 1 65	− 0 36	+ 1 15	− 0 46	1 24	248	3	227	− 21	− 20
1872 5	+ 2 15	− 0 12	+ 1 65	− 0 22	1 66	262	4	248	− 14	− 12
1873 5	+ 1 91	+ 0 16	+ 1 41	+ 0 06	1 41	272	4	269	− 3	− 1
1874 5	+ 1 92	+ 0 60	+ 1 42	+ 0 50	1 50	289	4	289	0	+ 4

The old and new series of observations agree well in giving for the value of the coefficient of this term,

The old series, $\alpha = 1'' \, 66$

The new series, $\alpha = 1'' \, 55$

The effect of the accidental errors will be, on the whole, to increase the value of the coefficient. I consider therefore that the value

$$\alpha = 1'' \, 50$$

may be adopted as the most probable which can be derived from all the observations

If we subtract, from each value of h and k in the preceding table, the periodic portions

$$h' = -1''.50 \sin [163^\circ.2 + 21^\circ.6 (t - 1868.5)]$$
$$k' = \quad 1''.50 \cos [163^\circ.2 + 21^\circ.6 (t - 1868.5)]$$

and take the mean value of the outstanding remainder for each series of observations we find it to be as follows

Old series, $h_0 = +0''.33$, $k_0 = -0''.17$
New series, $h_0 = +0''.65$, $k_0 = +0''.36$

The differences, $0''.01$ and $0''.08$, between these last values and those found on page 23 arise from the different value of the periodic term. I consider that the results of the second series are entitled to three times the weight of those of the first, and shall therefore put for the definitive values of h and k,

$$h = +0''.57 + h'$$
$$k = +0''.23 + k'$$

The corresponding corrections to the eccentricity and longitude of perigee are

$$\delta e = -0''.29$$
$$e\,\delta\pi = +0''.12$$
$$\delta\pi = +2''.2$$

The corrections to the moon's longitude are

$$\delta l = -h \sin g - k \cos g$$
$$= -0''.57 \sin g - 0''.23 \cos g + 1''.50 \sin (g + N - 90^\circ)$$

The last term is the hitherto-unsuspected inequality indicated by observations, but not yet known to be given by theory. It may be either an inequality of the eccentricity and perigee having a period of about $16\frac{2}{3}$ years, or one of the moon's mean longitude having a period of

$$27^d.4304 \pm 0^d.0040$$

Substituting first for N, and then for g, their values in terms of the time, the expression for the inequality of longitude becomes

$$1''.50 \sin [g + 73^\circ.2 + 21^\circ.6 (t - 1868.5)] = 1''.50 \sin (56^\circ.8 + 13^\circ.12413\, \tau),$$

τ being the time in days counted from Greenwich mean noon of 1850, Jan. 0.

It would perhaps be premature to introduce so purely empirical a term as this into lunar tables for permanent use, but where, as at present, it is requisite to obtain the corrections to the tables during a limited period with all possible accuracy, the evidence in favor of the reality of the term seems strong enough to justify its introduction. The only apparent cause to which the term can be attributed is the attraction of some one of the planets.

In the investigation of corrections to the longitude, it only remains to determine the slowly-varying corrections to the mean longitude, or to $n\,\delta z$, given by the observations. To determine the errors of short period, we have applied several corrections to the residuals, not as real, but only to render the various observations comparable. We

have now to consider the pure results of observations as they would have been had these corrections not been applied. These for the second series of observations are found by taking the sum of (1) the mean of the small corrections, applied on account of observatory and limb, to compensate for the systematic differences between results from different limbs or different observatories; (2) general corrections to make the residuals in the mean very small; (3) remaining outstanding correction found by solving the equations of condition.

The corrections from both series are as follow: the corrections since 1862 may be very closely represented by a term increasing uniformly with the time, as is shown by the last two columns.

First series.

Date.	$n\,\delta z$	Date.	$n\,\delta z$
	$''$		$''$
1847.8	− 0.15	1853.5	+ 1.77
1848.9	− 0.43	1854.6	+ 1.40
1850.1	+ 0.32	1855.8	+ 1.24
1851.2	+ 1.13	1856.9	+ 1.50
1852.4	+ 0.93	1858.1	+ 2.40

Second series.

Year.	(1)	(2)	(3)	$n\,\delta z$	$a + b\,t$	Δ
	$''$	$''$	$''$	$''$	$''$	$''$
1862.5	+ 0.45	+ 2.10	+ 0.04	+ 2.59	+ 1.52	+ 1.07
1863.5	+ 0.45	+ 1.20	− 0.27	+ 1.38	+ 0.60	+ 0.78
1864.5	0.00	0.00	− 0.49	− 0.49	− 0.32	− 0.17
1865.5	− 0.15	− 1.15	− 0.62	− 1.92	− 1.24	− 0.68
1866.5	− 0.15	− 2.00	− 0.75	− 2.90	− 2.16	− 0.74
1867.5	− 0.15	− 3.40	− 0.41	− 3.96	− 3.08	− 0.88
1868.5	− 0.15	− 4.05	− 0.20	− 4.40	− 4.00	− 0.40
1869.5	+ 0.08	− 4.85	− 0.21	− 4.98	− 4.92	− 0.06
1870.5	+ 0.08	− 5.50	− 0.09	− 5.51	− 5.84	+ 0.33
1871.5	0.00	− 6.35	− 0.52	− 6.87	− 6.76	− 0.11
1872.5	− 0.15	− 7.25	− 0.22	− 7.62	− 7.68	+ 0.06
1873.5	0.00	− 8.30	+ 0.10	− 8.20	− 8.60	+ 0.40
1874.5	0.00	− 9.45	+ 0.38	− 9.07	− 9.52	+ 0.45

§ 2

INVESTIGATION OF THE POLAR DISTANCE AND LATITUDE

It is a singular circumstance that during the last six years, at least, the observations of the moon's polar distance are much less accurate than those of its right ascension Whether this is to be attributed to the instruments, or whether it is a result of great irregularities in the outline of the lunar globe in the polar regions, cannot at present be decided To whatever cause we attribute the errors, their existence renders a rigorous treatment of the individual observations of little value We shall therefore, from the whole of the errors in declination, seek to obtain the best corrections to the inclination and node of the moon's orbit

From the derivatives of the moon's declination relatively to its true longitude, the inclination, and the node, which have already been given, we obtain

$$\delta\delta = \frac{d\delta}{dl}\,\delta l + \frac{d\delta}{dO}\,\delta O + \frac{d\delta}{di}\,\delta i$$

δl being known from the data already given, the equations of condition will be thrown into the form

$$\frac{d\delta}{i\,dO}\,i\,\delta O + \frac{d\delta}{di}\,\delta i = \delta\delta - \frac{d\delta}{dl}\,\delta l$$

From the numerical expressions already given, we have

$$\frac{d\delta}{dl}\,\delta l = \sec\delta\,[(0\,40 + 0\,08\cos O)\cos l + 0\,08\sin O\sin l]\,\delta l$$

If we put

$\delta\lambda =$ the correction to the moon's mean longitude,
$K = 0\,40 + 0\,08\cos O,$
$H = 0\,08\sin O,$

we have the quantities of the first order, with respect to the eccentricities,

$$\frac{d\delta}{d\lambda} = [K\cos l + H\sin l]\,[1 + 2\,e\cos(\lambda - \pi)\,]\sec\delta$$

The largest terms in sec δ are

$$1\,040 + 016\cos O - 040\cos 2\lambda - 016\cos(2\lambda - O),$$

while, if we replace l by the mean longitude, λ, we shall have

$$l = \lambda + 2\,e\sin(\lambda - \pi)$$
$$\sin l = \sin\lambda + e\sin(2\,\lambda - \pi) - e\sin\pi$$
$$\cos l = \cos\lambda + e\cos(2\,\lambda - \pi) - e\cos\pi$$

If we substitute these various quantities in the expression for $\frac{d\delta}{dl}\,\delta l$, we shall find

no sensible terms depending on the sine or cosine of the argument of latitude, $\lambda - \theta$. If we substitute for δl its value in $\delta\lambda$, we shall find the principal terms in $\cos\delta \frac{d\delta}{dl}\frac{dl}{d\lambda}$ to be

$$K\cos\lambda + H\sin\lambda + 3\,e\,K\cos(2\,\lambda - \pi) + 3\,e\,H\sin(2\,\lambda - \pi)$$

In consequence of the great number of revolutions of the moon through which the observations now under discussion extend, I have considered that all except the first two terms might be treated as accidental errors, which would cancel each other during the course of the observations. Using for $\delta\lambda$ the mean corrections to the moon's longitude, we have the following values of the correction to the declination for those errors of longitude:

Year	Correction	
	$''$	$''$
1862,	$+0.9\cos l$	$-0.2\sin l$
1863,	$+0.6$	-0.1
1864,	-0.1	0.0
1865,	-0.6	$+0.1$
1866,	-0.8	0.0
1867,	-0.1	-0.1
1868,	-1.4	-0.2
1869,	-1.8	-0.3
1870,	-2.2	-0.4
1871,	-2.8	-0.6
1872,	-3.3	-0.6
1873,	-3.8	-0.5
1874,	-4.2	-0.4

The mean correction to the moon's tabular north-polar distance for each year, from observations of each limb at each observatory, was taken with a view of detecting any constant error of sufficient magnitude to affect the final results for errors of the node and inclination. These means should have been taken after the application of the corrections just found: actually, however, they are the mean corrections given by the observations, after applying the following constant corrections to reduce the declinations to the same fundamental standard:

To Greenwich observations of N. P. D.	$''$
1862–67,	-0.4
1868–74,	$+0.2$

To Washington observations of N. P. D.	$''$
1862–65,	$+0.5$
1866–67,	-1.1
1868,	-1.2
1869,	-0.6
1870–72,	-0.4
1873–74,	0.0

These corrections are approximately those necessary to reduce the star-observations of the several years to Auwers's standard of declination. The change in the Greenwich correction between 1867 and 1868 probably arises from the introduction of a new

constant of refraction in 1868, while the change in the Washington correction in 1866 corresponds to the introduction of the large transit circle in place of the old mural circle.

Year	Correction to N. P. D. given by—			
	Greenwich		Washington	
	N. L.	S. L.	N. L.	S. L.
	$''$	$''$	$''$	$''$
1862	− 0.1	− 0.8	− 0.3	− 0.8
1863	+ 0.2	− 0.9	− 0.5	− 1.1
1864	+ 0.4	− 0.6	+ 0.8	− 0.9
1865	+ 0.5	− 0.2	+ 1.2	− 0.2
1866	− 0.7	− 0.3	+ 1.4	− 0.6
1867	− 0.4	− 0.6	+ 0.1	− 1.1
1868	− 0.7	− 1.0	+ 0.2	+ 0.2
1869	− 0.1	− 0.6	− 0.8	− 1.7
1870	− 0.6	− 0.1	− 0.1	− 1.8
1871	− 0.2	− 0.8	+ 2.1	− 1.8
1872	0.0	0.0	− 0.7	− 0.8
1873	− 0.9	+ 0.1	+ 2.0	− 0.1
1874			− 1.7	− 0.5

The large residuals of the Washington observations of the south limb led to the application of the further systematic correction of $+1''.0$ to all those observations before combining them all. The corrections arising from the error of mean longitude were then applied, and the outstanding residuals were considered to arise from accidental errors and from errors of the inclination and node. The equations of condition thus become

$$0.92 \sec \delta \, [\sin (l - \theta) \, \delta i - \cos (l - \theta) \, i \, \delta\theta] = \delta\delta$$

or

$$\sin (l - \theta) \, \delta i - \cos (l - \theta) \, i \, \delta\theta = 1.09 \cos \delta \times \delta\delta$$

Owing to the smallness of the final residuals, $\delta\delta$, the factor $1.09 \cos \delta$ may be considered as a constant, and, in the actual solution, has been put equal to unity. Its mean value is more exactly 1.04, and its effect may be obtained by dividing the final results by this factor.

The final values of the residuals were then arranged according to the values of $\lambda - \theta$, or the moon's mean argument of latitude, as the residuals in right ascension were arranged according to the mean anomaly. The sum of the residuals corresponding to each interval of 20° in the argument, with the corresponding number of observations for each year, is shown in the following table:

Sums of errors of the moon's corrected declination, given by observations at Greenwich and Washington

Limits of ?	1862		1863		1864		1865		1866		1867		1868	
	Σ δδ	N	Σ δδ	N	Σ δδ	N	Σ δδ	N	Σ δδ	N	Σ δδ	N	Σ δδ	N
° °	″		″		″		″		″		″		″	
0 to 20	− 3 8	8	+ 1 3	3	+ 4 0	8	+ 5 4	9	+26 7	11	− 2 5	8	+ 0 4	9
20 to 40	+ 9 6	9	+ 5 8	7	− 0 4	9	+ 6 0	7	+ 2 6	12	− 2 3	9	− 6 1	17
40 to 60	− 1 4	9	+ 6 9	10	+ 6 5	6	+ 9 7	7	+ 4 0	9	− 4 9	10	− 7 9	15
60 to 80	0 0	7	+16 4	10	+ 6 6	8	+ 8 7	12	− 1 1	16	+14 5	11	− 0 3	5
80 to 100	+ 8 6	11	+ 0 4	12	+11 1	6	+ 7 9	11	+ 5 5	7	+ 0 5	10	−12 8	12
100 to 120	+ 3 2	7	+ 8 5	15	+ 3 2	5	+ 7 4	7	− 1 0	8	− 6 1	6	+ 0 2	6
120 to 140	− 9 2	12	+ 3 1	8	− 6 1	8	+ 0 8	11	+11 9	14	−12 4	8	− 6 8	11
140 to 160	− 0 3	4	− 4 6	9	− 2 2	5	− 9 7	15	− 1 2	10	− 7 7	12	+ 6 2	14
160 to 180	+ 0 5	9	−10 4	6	−10 4	12	+ 0 5	9	+ 2 2	10	− 8 9	9	−11 2	9
180 to 200	− 8 6	6	− 5 7	11	− 0 6	7	− 5 3	12	− 7 7	6	−15 2	14	+ 3 1	10
200 to 220	−22 3	8	−11 6	10	+ 4 7	12	− 5 4	9	− 3 3	10	− 6 8	14	−11 8	11
220 to 240	−14 4	12	−10 2	9	− 8 8	10	− 1 0	7	− 2 0	13	− 5 9	12	−10 0	13
240 to 260	−12 4	7	−12 3	9	− 4 1	8	+ 4 6	11	+ 1 2	9	− 0 6	9	− 9 2	15
260 to 280	− 2 3	4	− 3 2	4	− 8 5	8	+ 1 5	9	− 5 3	9	+ 1 0	8	+ 1 9	9
280 to 300	− 2 2	7	− 4 3	8	− 8 4	11	− 4 0	4	− 3 5	9	−11 4	13	0 0	13
300 to 320	− 7 1	10	− 6 2	10	+ 9 6	8	+ 3 1	5	− 0 1	13	− 8 4	10	+ 0 4	8
320 to 340	+ 2 0	7	+ 3 4	8	+ 6 0	12	+ 8 6	6	+ 8 4	11	+ 2 1	9	− 6 7	14
340 to 360	+ 7 3	5	− 6 5	5	+ 4 9	13	+11 6	8	+ 7 0	14	+ 2 9	3	− 3 1	12
	−84 0	142	−75 0	154	−49 5	156	−25 4	159	−25 2	191	−93 1	175	−85 9	203
	+31 2		+45 8		+56 6		+ 75 8		+69 5		+21 0		+12 2	
	−52 8		−29 2		+ 7 1		+50 4		+44 3		−72 1		−73 7	

Sums of errors of the moon's corrected declination, &c—Continued

Limits of λ	1869		1870		1871		1872		1873		1874	
	$\Sigma\delta\delta$	N	$\Sigma\delta\delta$	N	$\Sigma\delta\delta$	N	$\Sigma\delta\delta$	N	$\Sigma\delta\delta$	N	$\Sigma\delta\delta$	N
° °	"		"		"		"		"		"	
0 to 20	+ 7 1	8	+ 3 7	7	− 3 8	5	− 8 0	6	+ 9 0	9	+ 7 7	12
20 to 40	+ 11 2	8	+ 6 6	10	− 0 1	4	− 7 0	6	− 7 7	10	− 7 5	7
40 to 60	+ 6 4	7	+ 8 6	10	− 0 8	11	− 1 2	10	− 2 8	4	− 17 0	11
60 to 80	− 5 0	9	+ 3 5	7	+ 13 2	11	− 5 1	7	− 13 7	7	− 25 4	14
80 to 100	− 2 0	11	+ 6 2	9	+ 13 1	9	− 3 7	8	+ 4 8	9	− 5 3	6
100 to 120	− 13 7	7	− 6 2	12	− 6 3	9	− 2 0	8	− 1 0	9	− 22 4	12
120 to 140	− 11 4	9	+ 4 5	7	− 1 9	8	+ 0 2	14	− 2 2	4	+ 2 1	6
140 to 160	− 15 4	12	− 11 7	11	− 4 6	7	− 8 9	9	+ 7 4	5	− 12 6	11
160 to 180	− 2 5	11	− 5 7	13	− 5 1	9	− 4 6	12	− 3 9	7	− 5 9	10
180 to 200	− 5 4	9	− 0 5	6	+ 5 4	11	− 6 0	3	− 8 6	7	− 7 2	8
200 to 220	− 5 4	4	− 10 2	12	− 6 2	10	− 2 9	9	+ 5 2	7	− 19 3	12
220 to 240	− 6 6	6	− 1 1	9	+ 9 1	14	− 4 5	10	− 2 7	5	− 15 2	6
240 to 260	− 18 4	12	− 11 1	8	+ 5 6	8	+ 13 7	13	+ 14 8	12	− 4 6	6
260 to 280	− 7 7	7	− 15 4	15	− 6 2	7	+ 2 2	11	+ 20 7	9	− 2 5	6
280 to 300	− 11 4	13	− 10 1	5	+ 3 6	8	+ 3 8	9	+ 3 3	10	− 5 5	13
300 to 320	+ 5 3	7	− 9 1	7	+ 3 8	8	− 1 3	11	+ 4 3	8	+ 0 3	10
320 to 340	+ 5 7	5	− 10 3	12	− 5 8	9	− 12 2	12	+ 14 0	7	+ 4 0	10
340 to 360	0 0	10	− 1 2	6	− 6 3	5	+ 0 3	9	+ 7 2	11	− 3 9	9
	− 104 9	155	− 92 6	166	− 47 1	153	− 67 4	167	− 42 6	140	− 154 3	169
	+ 35 7		+ 33 1		+ 53 8		+ 20 2		+ 90 7		+ 14 1	
	− 69 2		− 59 5		+ 6 7		− 47 2		+ 48 1		− 140 2	

The general irregularity of the residuals in declination is such that no great advantage would result in a separate solution of the equations for the separate years The sum of the residuals for each 20° of the argument was therefore taken during the whole thirteen years of observation, with the following result

$\lambda - \theta$	$\Sigma\Delta\delta$	N	$\lambda - \theta$	$\Sigma\Delta\delta$	N
° °	"		° °	"	
0 to 20	+ 47 2	103	180 to 200	− 62 3	110
20 to 40	+ 10 7	115	200 to 220	− 95 3	128
40 to 60	+ 6 1	119	220 to 240	− 73 3	126
60 to 80	+ 12 3	124	240 to 260	− 32 8	127
80 to 100	+ 34 3	121	260 to 280	− 23 8	106
100 to 120	− 36 2	111	280 to 300	− 50 1	123
120 to 140	− 27 4	120	300 to 320	− 5 4	115
140 to 160	− 65 3	124	320 to 340	+ 19 2	122
160 to 180	− 65 4	126	340 to 360	+ 20 2	110

Leaving in the equations a constant term δp, representing the mean constant error still outstanding in the measures of declination, the solution of the equations of condition given by the residuals gives the following results

$$\Delta p = -0''\ 17$$
$$\Delta i = +0''\ 15$$
$$i\Delta\theta = -0''\ 40$$

or,

Correction to the inclination, $-0''\ 15$

Correction to the longitude of node, $+4''\ 5$

This correction to the longitude of the node from Hansen's tables implies a diminution of the secular retrograde motion of the node, which is quite accordant with the results derived from ancient eclipses Hansen remarks that an increase of $12''$ per century in the longitude of the moon's node will improve the agreement of his tables with ancient eclipses,* and, if we suppose the tabular longitude of the node to have been correct in 1825, this would imply a correction of $+5''\ 2$ to the longitude of the node in 1868

* Darlegung, etc, Th ii, p 391

§ 3

AUXILIARY TABLES FOR FACILITATING THE COMPUTATION OF THE CORRECTIONS TO HANSEN'S "TABLES DE LA LUNE", GIVEN BY THE PRECEDING DISCUSSION

The following is a summary of the corrections to the longitude of the moon from Hansen's tables given by the preceding discussion The first six terms are applicable to the disturbed mean longitude, or "*Argument fondamental*", the remainder to the true longitude, but they may all be used as corrections of the "*Argument fondamental*" without serious error

Corrections on account of diminution of the solar parallax.. $n\,\delta z = +0''\,96 \sin D$
$+0''\,07 \sin (D-g)$
$-0''\,13 \sin (D+g')$

On account of hypothesis (here provisionally set aside), that the moon's center of gravity does not coincide with the center of figure, together with the correction to the evection resulting from the correction to the eccentricity $n\,\delta z = +0''\,09 \sin g'$
$-0''\,33 \sin 2D$
$-0''\,21 \sin (2D-g)$

On account of term accidentally introduced into the tables with a wrong sign........... $\delta v = -0''\,62 \sin (2g - 4g' + 2\omega - 4\omega')$

On account of correction to the eccentricity and perigee found from observations during 1847–74....... $\delta v = -0''\,57 \sin g - 0''\,23 \cos g$
$= 0''\,62 \sin (g + 202°\,0)$

Empirical term, necessary to satisfy observations, but not verified by theory .... $+1''\,50 \sin [g + 21°\,6\,(Y - 1865\,1)]$

Unexplained correction to the mean longitude, changing slowly from year to year.. See Table IV

The deduction of all these terms, except the last, has been fully given in the preceding pages This secular correction to the mean longitude has been derived from the outstanding errors of mean longitude given on page 30, in the column $n\,\delta z$, by supposing this quantity to vary according to some simple law, which law changes when necessary, so as to satisfy the observations within the mean limits of their probable error An examination of Table IV shows, that, from 1848 0 to 1855 5, the correction is supposed to increase uniformly at the rate of 0″ 20 per annum It is then supposed to remain constant until nearly 1863 0, a period during which the observations are not continuous, there being no comparisons with theory from 1859 to 1861 inclusive From 1863 0 until the present time, the observations are well represented by the correction

$$-5''\,53 - 0''\,86\,(t - 1870\,0) + 0''\,02\,(t - 1870\,0)^2$$

The continuance of this correction beyond 1875 0 is, of course, purely conjectural

TABLES FOR APPLYING THE PRECEDING CORRECTIONS.

The following tables are designed to facilitate the computation of the corrections

just given. To avoid the necessity of referring to Hansen's tables, the values of all the necessary arguments are given for the years 1850 to 1889 in Tables I to III.

Table I: the epochs are January 0, Greenwich mean noon of common years, and January 1 of leap years. All the arguments increase uniformly by a unit in a day.

Argument g is the moon's mean anomaly, converted into days by dividing its expression in degrees by 13.065. It is equal to Hansen's argument g diminished by 15 days.

Argument D shows the number of days since mean new moon, or, it is the mean departure of the moon from the sun expressed in days. It is equal to Hansen's argument 33 diminished by 30 days, or, which amounts to the same thing, by $0^d.47$.

Argument A gives the number of days from the time when the angle

$$2g - 4g' + 2\omega - 4\omega'$$

was last zero.

Argument B is that of the empirical term indicated by observations, but not given by theory.

Argument u is that of latitude, or the number of days since the mean moon last passed her ascending node.

Tables II and III do not seem to need explanation. In using the former, care must be taken to diminish by one day the dates for January and February of leap years.

Table IV gives the secular corrections to the mean longitude, or to $n\delta z$, obtained from observations in the manner already described.

Table V, argument A, gives the correction for the term introduced into the tables with a wrong sign, described on page 9. It is properly to be applied to the true longitude, and is therefore designated as δv.

Table VI gives the empirical term, which, so far as is known, may be applied to the true longitude.

Table VII gives the sum of the terms of mean longitude

$$\begin{aligned} &+0''.96 \sin D \\ &-0''.33 \sin 2D \\ &-0''.13 \sin (D+g') \\ &+0''.09 \sin g' \end{aligned}$$

The sun's mean anomaly, g', having a period of a year, the sum of these terms can be expressed as a function of D and the month, and is given in the table for the middle of each month, and for each day of D.

Table VIII gives the sum of the terms of true longitude which depend wholly or partly on the moon's mean anomaly, namely:

$$\begin{aligned} &+0''.62 \sin (g+202^\circ.0) \\ &+0''.07 \sin (D-g) \\ &-0''.21 \sin (2D-g) \end{aligned}$$

The sum of the terms of $n\ \delta z$ are to be reduced to corrections of the longitude in orbit by multiplication by the factor

$$1 + 2e \cos g + \frac{5}{2} e^2 \cos 2g$$

This factor, less unity, is given in Table IX.

For convenience, the unit of the factor is omitted from the tabular numbers, so that it is only necessary to add the product $F \times n\ \delta z$ in with $n\ \delta z$ and δv to have the correction of the true longitude in orbit

These corrections being applied to the longitude of the moon's center from Hansen's tables, that longitude may be regarded as correct, excepting a small correction, which may probably be regarded as constant during any one period not exceeding six months, and which may be attributed to the adopted position of the equinox It will be best determined from occultations of stars observed at points whose longitudes from Greenwich are accurately known by telegraph, and will then be applicable to the determination of the longitude of any station from occultations

If the corrections here deduced are applied to the errors of the lunar ephemeris derived from meridian observations, it must be remembered that these observations are made on the moon's limb, while the corrections are applicable to the center Hence, the value of the moon's semi-diameter must, if great refinement is aimed at, be varied with the observer, the instrument, and the brightness of the sky For large instruments, Hansen's semi-diameter is about 1″ too great, even at night

The sum of all the terms of $n\ \delta z$, δv, and $F \times n\ dz$ from the tables will be the correction of the longitude in orbit This will not be rigorously the same as the correction to the ecliptic longitude

Table X gives the small factor $(F\ l)$ by which the orbit longitude must be increased or diminished to obtain the ecliptic longitude This factor may generally be disregarded

Table X also gives the data for the correction of the moon's latitude, namely, (1) a factor $(F\ \beta)$ by which the correction of the moon's argument of latitude must be multiplied, and (2) the term

$$\delta\beta_1 = -0''\ 15 \sin u$$

arising from the correction to the tabular inclination of the moon's orbit The correction of the moon's argument of latitude being that of her longitude, *minus* the correction of her node, the whole correction to the latitude will be

$$\delta\beta = \delta\beta_1 + (F\ \beta)\ (\delta v - 4''\ 5)$$

Table XI gives the factors for converting corrections of longitude and latitude into corrections of right ascension and declination The formulæ are

$$\delta\ \text{AR} = \delta v + (v\ \alpha)\ \delta v + (\beta\ \alpha)\ \delta\beta$$
$$\delta\ \text{Dec} = \delta\beta + (v\ \delta)\ \delta v + (\beta\ \delta)\ \delta\beta$$

The side argument is the moon's longitude, and in the coefficients $(v\ \alpha)$ and perhaps $(\beta\ \alpha)$ regard must be had to the moon's latitude also Three columns are therefore given for latitude, $-5°$, $0°$, and $+5°$ respectively

As an example of the use of the tables of corrections, we will commence the determination of the corrections for September, 1874. We find the values of the arguments for September 1, from Tables I to III, as follows:

	g	D	A	B	u
1874 . . .	5.4	12.1	8.0	20.0	1.9
Sept. 1 . .	23.6	7.8	1.9	24.6	26.2
Periods . .	−27.6	. .	. .	−27.4	−27.2
Arg. Sept. 1 .	1.4	19.9	9.9	17.2	0.9
Arg. Oct. 1 .	3.8	20.3	39.9 or 7.8	47.2 or 19.8	30.9 or 3.7

$$D = 19.9$$
$$g = 1.4$$
$$D - g = 18.5$$

The tabular numbers are then found as follows, with an argument increasing by unity each day. From Table VIII, we take a mean from columns 18 and 19.

September . .	1.	2.	3.	4.	5.	6.	7.	8.
	″	″	″	″	″	″	″	″
Table IV ($n\,\delta z$) .	− 9.11	− 9.11	− 9.11	− 9.11	− 9.12	− 9.12	− 9.12	− 9.12
V (δv) . .	+ 0.40	+ 0.55	+ 0.62	+ 0.59	+ 0.48	+ 0.28	+ 0.05	− 0.18
VI (δv) .	− 1.07	− 1.29	− 1.42	− 1.50	− 1.48	− 1.40	− 1.23	− 1.01
VII ($n\,\delta z$) .	− 1.29	− 1.25	− 1.12	− 0.95	− 0.76	− 0.56	− 0.37	− 0.22
VIII (δv) .	− 0.65	− 0.72	− 0.77	− 0.78	− 0.75	− 0.69	− 0.60	− 0.48
	−11.72	−11.82	−11.80	−11.75	−11.63	−11.49	−11.27	−11.01
$n\,\delta z \times F$, Table IX	− 1.12	− 0.93	− 0.71	− 0.47	− 0.28	− 0.06	+ 0.18	+ 0.37
δv	−12.84	−12.75	−12.51	−12.22	−11.91	−11.55	−11.09	−10.64
$\delta v - 4''.5$	−17.3	−17.2	−17.0	−16.7	−16.4	−16.0	−15.6	−15.1
Table X (F . β) .	+ 0.088	+ 0.082	+ 0.070	+ 0.056	+ 0.038	+ 0.019	− 0.002	− 0.022
X ($\delta\beta_1$) . .	− 0.03	− 0.07	− 0.10	− 0.12	− 0.14	− 0.15	− 0.15	− 0.14
($\delta v - 4''.5$)(F. β) .	− 1.52	− 1.38	− 1.19	− 0.93	− 0.63	− 0.30	+ 0.03	+ 0.33
$\delta\beta$	− 1.55	− 1.45	− 1.29	− 1.05	− 0.77	− 0.45	− 0.12	+ 0.19
	°	°	°	°	°	°	°	°
☽'s longitude .	46.5	60.7	74.6	88.1	101.5	114.5	127.4	140.0
	″	″	″	″	″	″	″	″
$(1+(v\,.\,a))\;\delta v$ . .	−13.08	−13.49	−13.84	−13.81	−13.39	−12.61	−11.62	−10.67
$(\beta\,.\,a)\;\delta\beta$ . . .	+ 0.47	+ 0.32	+ 0.16	+ 0.02	− 0.07	− 0.09	− 0.03	+ 0.06
δAR	−12.6	−13.2	−13.7	−13.8	−13.5	−12.7	−11.6	−10.6
	− 0s.84	− 0s.88	− 0s.91	− 0s.92	− 0s.90	− 0s.85	− 0s.78	− 0s.71
	″	″	″	″	″	″	″	″
$(v\,.\,\delta)\;\delta v$ . . .	− 3.70	− 2.64	− 1.41	− 0.18	+ 1.02	+ 2.05	+ 2.84	+ 3.35
$(1+\beta\,.\,\delta)\;\delta\beta$ . .	− 1.50	− 1.42	− 1.28	− 1.05	− 0.77	− 0.44	− 0.12	+ 0.18
δDec.	− 5.2	− 4.1	− 2.7	− 1.2	+ 0.2	+ 1.6	+ 2.7	+ 3.5

This computation has been continued to 1875, January 31, and the results are shown in the following table

Corrections to the Ephemeris derived from Hansen's Tables of the Moon, for Greenwich mean noon of each day, from 1874, September 1, to 1875, January 31

Date Gr mean noon	Correction to tabular— Long	Lat	R A	Dec	Date Gr mean noon	Correction to tabular— Long	Lat	R A	Dec
1874	′	″	″	″	1874	″	″	″	′
Sept 1	−12 8	− 1 6	−12 6	− 5 2	Oct 11	− 7 5	+ 1 1	− 6 7	+ 3 6
2	12 8	1 5	13 2	4 1	12	7 2	1 0	6 8	3 2
3	12 5	1 3	13 7	2 7	13	6 9	1 0	6 8	2 7
4	12 2	1 0	13 8	− 1 2	14	6 6	0 8	7 0	2 0
5	11 9	0 8	13 5	+ 0 2	15	6 4	0 7	7 0	1 3
6	−11 6	− 0 5	−12 7	+ 1 6	16	− 6 2	+ 0 5	− 7 0	+ 0 6
7	11 1	− 0 1	11 6	2 7	17	6 1	0 3	6 9	− 0 2
8	10 6	+ 0 2	10 6	3 5	18	6 2	+ 0 1	6 8	1 0
9	10 1	0 5	9 6	4 1	19	6 4	− 0 1	6 6	1 8
10	9 6	0 7	8 6	4 3	20	6 8	0 4	6 6	2 5
11	− 9 0	+ 0 9	− 7 9	+ 4 4	21	− 7 5	− 0 6	− 6 9	− 3 3
12	8 3	1 0	7 3	4 2	22	8 3	0 9	7 3	4 1
13	7 6	1 1	6 7	3 8	23	9 3	1 1	8 1	4 7
14	6 8	1 0	6 2	3 3	24	10 4	1 3	9 2	5 2
15	6 2	1 0	5 9	2 8	25	11 4	1 5	10 6	5 2
16	− 5 6	+ 0 9	− 5 7	+ 2 2	26	−12 4	− 1 5	−12 2	− 4 8
17	5 2	0 7	5 5	1 6	27	13 2	1 4	13 9	3 8
18	5 0	0 6	5 5	1 0	28	13 6	1 2	15 1	2 3
19	5 1	0 4	5 8	+ 0 3	29	13 8	1 0	15 7	− 0 6
20	5 4	+ 0 2	6 1	− 0 4	30	13 6	0 6	15 3	+ 1 1
21	− 6 1	0 0	− 6 6	− 1 2	31	−13 2	− 0 3	−14 2	+ 2 6
22	7 0	− 0 2	7 2	2 2	Nov 1	12 4	+ 0 1	12 6	3 6
23	8 1	0 5	7 8	3 2	2	11 4	0 4	11 1	4 3
24	9 4	0 8	8 5	[illegible] 3	3	10 5	0 7	9 6	4 5
25	10 6	1 1	9 3	5 2	4	9 5	0 8	8 4	4 5
26	−11 8	− 1 3	−10 3	− 5 8	5	− 8 5	+ 1 0	− 7 4	+ 4 3
27	12 7	1 5	11 5	6 0	6	7 7	1 0	6 7	3 9
28	13 4	1 6	12 8	5 7	7	7 1	1 0	6 3	3 5
29	13 7	1 5	13 9	4 8	8	6 6	1 0	6 2	3 1
30	13 8	1 4	14 9	3 4	9	6 4	1 0	6 2	2 6
Oct 1	−13 5	− 1 2	−15 2	− 1 7	10	− 6 3	+ 0 9	− 6 5	+ 2 1
2	13 0	0 9	14 7	− 0 1	11	6 3	0 7	6 9	1 5
3	12 2	0 5	13 6	+ 1 4	12	6 5	0 6	7 3	+ 0 8
4	11 5	− 0 2	12 2	2 6	13	6 8	0 4	7 7	− 0 1
5	10 6	+ 0 1	10 8	3 3	14	7 1	+ 0 1	7 8	1 0
6	− 9 9	+ 0 4	− 9 4	+ 3 9	15	− 7 4	− 0 1	− 7 8	− 1 8
7	9 2	0 6	8 4	4 1	16	7 7	0 4	7 7	2 7
8	8 7	0 8	7 7	4 2	17	8 1	0 6	7 6	3 4
9	8 2	1 0	7 2	4 2	18	8 4	0 8	7 6	4 0
10	7 8	1 0	6 8	4 0	19	8 9	1 0	7 8	4 5

Corrections to the Ephemeris derived from Hansen's Tables of the Moon, etc —Continued

Date Gr mean noon		Correction to tabular— Long	Lat	R A	Dec
1874		″	″	″	″
Nov	20	− 9 4	− 1 2	− 8 2	− 4 8
	21	10 0	1 3	9 0	4 8
	22	10 6	1 3	10 1	4 5
	23	11 2	1 3	11 4	3 8
	24	11 7	1 2	12 7	2 6
	25	−12 2	− 1 0	−13 8	− 1 2
	26	12 5	0 6	14 1	+ 0 5
	27	12 6	− 0 3	13 8	2 0
	28	12 5	0 0	13 0	3 3
	29	12 1	+ 0 4	11 9	4 3
	30	−11 7	+ 0 7	−10 9	+ 4 9
Dec	1	11 0	0 9	9 9	5 1
	2	10 3	1 1	9 0	5 1
	3	9 4	1 2	8 2	4 7
	4	8 5	1 2	7 5	4 2
	5	− 7 7	+ 1 1	− 7 0	+ 3 6
	6	7 0	1 0	6 8	3 0
	7	6 4	0 9	6 6	2 3
	8	6 1	0 8	6 5	1 6
	9	6 0	0 6	6 7	0 9
	10	− 6 1	+ 0 4	− 6 9	+ 0 1
	11	6 4	+ 0 2	7 1	− 0 7
	12	6 8	0 0	7 3	1 5
	13	7 4	− 0 3	7 5	2 4
	14	8 0	0 6	7 6	3 3
	15	− 8 7	− 0 8	− 7 8	− 4 0
	16	9 3	1 1	8 1	4 7
	17	9 9	1 2	8 6	5 0
	18	10 3	1 3	9 2	5 1
	19	10 7	1 4	10 0	4 8
	20	−11 0	− 1 3	−11 0	− 4 1
	21	11 2	1 2	12 0	3 1
	22	11 4	1 0	12 7	1 7
	23	11 4	0 7	12 9	− 0 2
	24	11 4	0 4	12 6	+ 1 3
	25	−11 3	− 0 1	−12 0	+ 2 6
	26	11 1	+ 0 3	11 1	3 6

Date Gr mean noon		Correction to tabular— Long	Lat	R A	Dec
1874		″	″	′	′
Dec	27	−10 8	+ 0 6	−10 2	+ 4 3
	28	10 4	0 8	9 5	4 7
	29	10 1	1 0	8 9	4 9
	30	9 6	1 2	8 4	4 9
	31	9 2	1 2	8 1	4 6
1875					
Jan	1	− 8 7	+ 1 2	− 7 9	+ 4 1
	2	8 2	1 1	7 9	3 5
	3	7 8	1 0	7 9	2 9
	4	7 4	0 9	7 9	2 1
	5	7 1	0 6	7 8	1 2
	6	− 6 9	+ 0 4	− 7 8	+ 0 4
	7	6 8	+ 0 2	7 7	− 0 5
	8	6 9	0 0	7 5	1 4
	9	7 2	− 0 3	7 4	2 2
	10	7 6	0 5	7 3	3 0
	11	− 8 0	− 0 7	− 7 3	− 3 7
	12	8 6	1 0	7 5	4 3
	13	9 2	1 2	8 0	4 7
	14	9 7	1 3	8 6	4 9
	15	10 3	1 4	9 5	4 8
	16	−10 8	− 1 3	−10 6	− 4 3
	17	11 2	1 2	11 7	3 4
	18	11 5	1 0	12 6	2 2
	19	11 6	0 8	13 1	− 0 8
	20	11 6	0 5	13 0	+ 0 8
	21	−11 5	− 0 1	−12 4	+ 2 2
	22	11 1	+ 0 2	11 4	3 3
	23	10 7	0 5	10 3	4 1
	24	10 2	0 7	9 3	4 5
	25	9 6	1 0	8 5	4 6
	26	− 9 1	+ 1 1	− 7 9	+ 4 6
	27	8 5	1 1	7 5	4 3
	28	8 1	1 1	7 2	4 0
	29	7 7	1 1	7 2	3 5
	30	7 5	1 0	7 4	2 9
	31	− 7 4	+ 0 9	− 7 7	+ 2 3

TABLES.

TABLES

Table I

Values of the Arguments for the beginning of each year

Year	g	D	A	B	u
1850	1 8	16 7	6 3	4 3	25 5
1851	8 6	27 3	0 0	12 7	9 5
1852 B	16 4	9 4	10 9	22 1	21 7
1853	23 1	20 0	4 7	3 1	5 8
1854	2 4	1 1	14 6	11 5	17 0
1855	9 2	11 8	8 4	19 9	1 0
1856 B	17 0	23 4	3 2	1 9	13 3
1857	23 8	4 5	13 1	10 3	24 5
1858	3 0	15 1	6 9	18 7	8 5
1859	9 8	25 8	0 7	27 1	19 8
1860 B	17 6	7 9	11 6	9 1	4 8
1861	24 4	18 5	5 3	17 5	16 0
1862	3 6	29 2	15 2	25 9	0 1
1863	10 4	10 2	9 0	6 9	11 3
1864 B	18 2	21 9	3 7	16 3	23 6
1865	25 0	3 0	13 7	24 7	7 6
1866	4 2	13 6	7 4	5 7	18 8
1867	11 0	24 3	1 2	14 1	2 8
1868 B	18 8	6 4	12 1	23 5	15 1
1869	25 6	17 0	5 9	4 4	26 3
1870	4 8	27 6	15 8	12 8	10 4
1871	11 6	8 7	9 6	21 3	21 6
1872 B	19 4	20 4	4 4	3 2	6 6
1873	26 2	1 5	14 3	11 6	17 9
1874	5 4	12 1	8 0	20 0	1 9
1875	12 2	22 7	1 8	1 0	13 1
1876 B	20 0	4 8	12 7	10 4	25 4
1877	26 8	15 5	6 5	18 8	9 1
1878	6 0	26 1	0 3	27 2	20 7
1879	12 8	7 2	10 2	8 2	4 7
1880 B	20 6	18 8	5 0	17 6	16 9
1881	27 4	29 4	14 9	26 0	1 0
1882	6 6	10 6	8 6	7 0	12 2
1883	13 4	21 2	2 4	15 4	23 4
1884 B	21 2	3 3	13 3	24 8	8 5
1885	0 5	13 9	7 1	5 8	19 7
1886	7 3	24 6	0 9	14 2	3 7
1887	14 0	5 7	10 8	22 6	15 0
1888 B	21 8	17 3	5 5	4 6	0 0
1889	28 6	27 9	15 4	13 0	11 2

Table II

Reduction of the Arguments to the zero-day of each month

Month		g	D	A	B	u
Jan	0*	0 0	0 0	0 0	0 0	0 0
Feb	0*	3 4	1 5	14 9	3 6	3 8
Mar	0	3 9	0 0	10 6	4 1	4 6
April	0	7 3	1 4	9 3	7 7	8 4
May	0	9 8	1 9	7 0	10 3	11 2
June	0	13 2	3 3	5 7	13 8	14 9
July	0	15 7	3 8	3 5	16 4	17 7
Aug	0	19 1	5 3	2 2	20 0	21 5
Sept	0	22 6	6 8	0 9	23 6	25 2
Oct	0	25 0	7 2	14 8	26 1	0 9
Nov	0	0 9	8 7	13 5	2 3	4 7
Dec	0	3 3	9 1	11 2	4 8	7 5

* In January and February of leap-years, the numbers taken from Table II are to be diminished by a unit

Table III

Periods of the Arguments.

	g	D	A	B	u
P	27 6	29 5	16 1	27 4	27 2
$2P$	55 1	59 1	32 3	54 9	54 4
$3P$	82 7	88 6	48 4	82 3	81 6
$4P$	110 2	118 1	64 6	109 7	108 8

Table IV

Secular Terms

Year	$n\,\delta z$	Diff
	"	"
1848 0	0 00	+ 0 20
1849 0	+ 0 20	0 20
1850 0	0 40	0 20
1851 0	0 60	0 20
1852 0	0 80	0 20
1853 0	1 00	+ 0 20
1854 0	+ 1 20	0 20
1855 0	1 40	+ 0 10
1856 0	1 50	0 00
1857 0	1 50	0 00
1858 0	1 50	0 00
1859 0	+ 1 50	0 00
1860 0	1 50	0 00
1861 0	1 50	0 00
1862 0	1 50	− 0 03
1863 0	1 47	− 1 12
1864 0	+ 0 35	− 1 08
1865 0	− 0 73	− 1 04
1866 0	− 1 77	− 1 00
1867 0	− 2 77	− 0 96
1868 0	− 3 73	− 0 92
1869 0	− 4 65	− 0 88
1870 0	− 5 53	− 0 84
1871 0	− 6 37	− 0 80
1872 0	− 7 17	− 0 76
1873 0	− 7 93	− 0 72
1874 0	− 8 65	− 0 68
1875 0	− 9 33	- 0 64
1876 0	− 9 97	− 0 60
1877 0	- 10 57	− 0 56
1878 0	−11 13	− 0 52
1879 0	−11 65	− 0 48
1880 0	−12 13	

Table V

Argument A

A	δv
	"
0	0 00
1	− 0 23
2	− 0 44
3	− 0 57
4	− 0 62
5	− 0 57
6	− 0 44
7	− 0 25
8	− 0 02
9	+ 0 22
10	0 42
11	0 56
12	0 62
13	0 58
14	+ 0 46
15	0 26
16	+ 0 03
17	− 0 20
18	− 0 42
19	− 0 56
20	− 0 62
21	− 0 59
22	− 0 47
23	− 0 28
24	− 0 05
25	+ 0 19
26	0 40
27	0 55
28	0 62
29	0 59
30	+ 0 49
31	0 30
32	+ 0 07
33	− 0 17
34	− 0 38
35	− 0 54
36	− 0 62
37	− 0 60
38	− 0 49
39	− 0 31
40	− 0 09
41	+ 0 15
42	0 38
43	0 54
44	0 61
45	0 60
46	+ 0 51
47	0 33
48	+ 0 10
49	− 0 14
50	− 0 36

Table VI

Argument B (Empirical Term)

B	δv	B	δv
	"		"
0	0 00	40	+ 0 39
1	+ 0 34	41	+ 0 05
2	0 66	42	− 0 29
3	0 95	43	− 0 62
4	1 19	44	− 0 91
5	+ 1 37	45	− 1 16
6	1 47	46	− 1 34
7	1 50	47	− 1 46
8	1 45	48	− 1 50
9	1 32	49	− 1 46
10	+ 1 13	50	− 1 34
11	0 88	51	− 1 16
12	0 57	52	− 0 91
13	+ 0 25	53	− 0 62
14	− 0 10	54	− 0 29
15	− 0 44	55	+ 0 05
16	− 0 75	56	0 39
17	− 1 03	57	0 71
18	− 1 25	58	0 99
19	− 1 40	59	1 22
20	− 1 49	60	+ 1 39
21	− 1 49	61	1 48
22	- 1 42	62	1 50
23	− 1 27	63	1 44
24	− 1 06	64	1 30
25	− 0 79	65	+ 1 08
26	− 0 48	66	0 83
27	− 0 15	67	0 53
28	+ 0 19	68	+ 0 19
29	0 53	69	− 0 15
30	+ 0 83	70	− 0 48
31	1 08	71	− 0 79
32	1 30	72	− 1 06
33	1 44	73	− 1 27
34	1 50	74	− 1 42
35	+ 1 48	75	− 1 49
36	1 39	76	− 1 49
37	1 22	77	− 1 40
38	0 99	78	− 1 25
39	0 71	79	− 1 03
40	+ 0 39	80	− 0 75

TABLE VII, $n\,\delta z$

Arguments, D and the month

D	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
	″	″	″	″	″	″	″	″	″	″	″	″
0	−0 01	−0 03	−0 04	−0 04	−0 03	−0 01	+0 01	+0 03	+0 04	+0 04	+0 03	+0 01
1	+0 03	+0 01	+0 02	+0 03	+0 05	+0 08	+0 10	+0 11	+0 11	+0 10	+0 07	+0 05
2	0 08	0 09	0 10	0 13	0 16	0 19	0 21	0 20	0 19	0 16	0 13	0 10
3	0 16	0 19	0 22	0 26	0 29	0 32	0 34	0 31	0 28	0 24	0 21	0 18
4	0 29	0 32	0 38	0 43	0 46	0 49	0 49	0 46	0 41	0 36	0 32	0 29
5	+0 45	+0 50	+0 56	+0 62	+0 66	+0 67	+0 67	+0 62	+0 56	+0 50	+0 46	+0 45
6	0 63	0 68	0 77	0 83	0 85	0 87	0 85	0 79	0 71	0 65	0 62	0 60
7	0 80	0 87	0 96	1 02	1 05	1 06	1 02	0 95	0 86	0 80	0 77	0 76
8	0 93	1 02	1 12	1 18	1 20	1 19	1 15	1 06	0 96	0 90	0 88	0 89
9	1 02	1 12	1 21	1 27	1 29	1 27	1 21	1 11	1 01	0 95	0 94	0 96
10	+1 04	+1 15	+1 25	+1 30	+1 30	+1 26	+1 18	+1 08	+0 98	+0 93	+0 93	+0 97
11	0 97	1 07	1 18	1 22	1 20	1 15	1 07	0 96	0 86	0 82	0 83	0 89
12	0 82	0 92	1 03	1 06	1 02	0 96	0 86	0 76	0 66	0 63	0 66	0 72
13	0 58	0 68	0 77	0 79	0 74	0 66	0 56	0 46	0 37	0 35	0 40	0 48
14	+0 28	0 39	0 47	0 48	0 42	+0 32	+0 22	+0 12	+0 04	+0 03	+0 09	+0 18
15	−0 02	+0 08	+0 14	+0 14	+0 07	−0 03	−0 13	−0 23	−0 30	−0 30	−0 22	−0 12
16	−0 34	−0 25	−0 20	−0 22	−0 29	−0 40	−0 50	−0 59	−0 61	−0 62	−0 55	−0 44
17	−0 60	−0 53	−0 49	−0 52	−0 61	−0 71	−0 81	−0 89	−0 92	−0 89	−0 81	−0 70
18	−0 81	−0 74	−0 72	−0 76	−0 86	−0 97	−1 05	−1 12	−1 14	−1 00	−1 00	−0 89
19	−0 93	−0 88	−0 88	−0 89	−1 02	−1 13	−1 21	−1 26	−1 23	−1 22	−1 12	−1 01
20	−0 97	−0 94	−0 94	−1 00	−1 10	−1 21	−1 27	−1 30	−1 30	−1 24	−1 14	1 03
21	−0 93	−0 92	−0 93	−0 99	−1 09	−1 18	−1 24	−1 25	−1 23	−1 17	−1 08	−0 99
22	−0 83	−0 83	−0 86	−0 92	−1 01	−1 09	−1 13	−1 13	−1 10	−1 04	−0 95	−0 87
23	−0 69	−0 70	−0 72	−0 78	−0 87	−0 94	−0 96	−0 95	−0 92	−0 86	−0 78	−0 67
24	−0 51	−0 54	−0 59	−0 64	−0 71	−0 77	−0 79	−0 77	−0 72	−0 67	−0 60	−0 55
25	−0 37	−0 39	−0 43	−0 48	−0 54	−0 59	−0 59	−0 56	−0 52	−0 47	−0 41	−0 37
26	−0 23	−0 27	−0 31	−0 36	−0 40	−0 41	−0 41	−0 38	−0 34	−0 29	−0 25	−0 23
27	−0 14	−0 17	−0 20	−0 24	−0 27	−0 28	−0 26	−0 23	−0 20	−0 16	−0 13	−0 12
28	−0 07	−0 11	−0 12	−0 15	−0 16	−0 16	−0 14	−0 10	−0 09	−0 06	−0 05	−0 05
29	−0 03	−0 06	−0 07	−0 08	−0 08	−0 06	−0 04	−0 02	0 00	+0 01	0 00	−0 01
30	0 00	−0 01	−0 02	−0 01	0 00	+0 02	+0 04	+0 06	+0 07	0 06	+0 05	+0 02
31	+0 05	+0 05	+0 07	+0 08	+0 10	0 13	0 15	0 16	0 14	0 13	0 11	0 07
32	+0 12	+0 14	+0 16	+0 19	+0 22	+0 26	+0 27	+0 26	+0 23	+0 20	+0 17	+0 14

NOTE.—Each column is computed for the middle of the month, but may be used for the entire month without an error ever exceeding 0″.05. If much greater accuracy than this is required, a horizontal interpolation must be used.

TABLE VIII, δv

Horizontal Argument, or Argument at top, $D-g$, or $D-g+30$ Vertical Argument, g

g	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	′	″	″	″	″	″	″	″	″	″	″	″	″	″	″
0	−0 23	−0 30	−0 36	−0 39	−0 39	−0 35	−0 28	−0 19	−0 11	−0 03	+0 02	+0 03	0 00	−0 06	−0 15
1	−0 39	−0 46	−0 50	−0 53	−0 50	−0 44	−0 36	−0 28	−0 19	−0 12	−0 09	−0 09	−0 14	−0 22	−0 31
2	−0 54	−0 60	−0 64	−0 62	−0 58	−0 52	−0 43	−0 34	−0 25	−0 20	−0 19	−0 21	−0 27	−0 35	−0 45
3	−0 66	−0 71	−0 72	−0 69	−0 64	−0 56	−0 47	−0 37	−0 31	−0 27	−0 26	−0 31	−0 38	−0 47	−0 58
4	−0 75	−0 77	−0 77	−0 74	−0 67	−0 58	−0 48	−0 39	−0 34	−0 31	−0 33	−0 38	−0 46	−0 57	−0 67
5	−0 79	−0 81	−0 80	−0 74	−0 66	−0 55	−0 46	−0 39	−0 34	−0 34	−0 36	−0 43	−0 53	−0 64	−0 74
6	−0 80	−0 81	−0 76	−0 70	−0 60	−0 50	−0 42	−0 35	−0 33	−0 33	−0 38	−0 47	−0 57	−0 67	−0 76
7	−0 78	−0 76	−0 71	−0 62	−0 52	−0 43	−0 34	−0 30	−0 28	−0 31	−0 38	−0 47	−0 57	−0 67	−0 73
8	−0 71	−0 68	−0 61	−0 51	−0 2	−0 32	−0 26	−0 22	−0 22	−0 27	−0 35	−0 45	−0 54	−0 61	−0 68
9	−0 62	−0 56	−0 47	−0 38	−0 28	−0 20	−0 14	−0 12	−0 15	−0 21	−0 30	−0 38	−0 47	−0 53	−0 58
10	−0 49	−0 41	−0 33	−0 22	−0 13	−0 06	−0 02	−0 02	−0 06	−0 13	−0 22	−0 30	−0 37	−0 44	−0 48
11	−0 33	−0 26	−0 16	−0 06	+0 03	+0 09	+0 11	+0 10	+0 04	−0 03	−0 12	−0 19	−0 28	−0 33	−0 33
12	−0 18	−0 08	+0 02	+0 11	0 19	0 24	0 25	0 21	0 15	+0 07	−0 01	−0 10	−0 17	−0 18	−0 18
13	0 00	+0 10	0 19	0 28	0 35	0 38	0 37	0 33	0 26	0 19	+0 09	0 00	−0 03	−0 04	−0 01
14	+0 17	0 26	0 36	0 45	0 50	0 51	0 49	0 44	0 37	0 27	0 17	+0 13	+0 10	+0 11	+0 14
15	+0 32	+0 42	+0 52	+0 59	+0 63	+0 62	+0 59	+0 53	+0 43	+0 34	+0 28	+0 23	+0 23	+0 24	+0 29
16	0 46	0 56	0 65	0 71	0 73	0 71	0 66	0 58	0 48	0 42	0 35	0 33	0 33	0 37	0 43
17	0 58	0 68	0 75	0 80	0 79	0 76	0 68	0 60	0 53	0 46	0 42	0 41	0 43	0 48	0 56
18	0 67	0 76	0 82	0 84	0 82	0 77	0 69	0 62	0 54	0 49	0 46	0 47	0 50	0 57	0 65
19	0 73	0 80	0 83	0 84	0 81	0 75	0 67	0 59	0 53	0 48	0 48	0 49	0 56	0 63	0 72
20	+0 74	+0 79	+0 81	+0 80	+0 76	+0 69	+0 61	+0 54	+0 49	+0 47	+0 46	+0 51	+0 58	+0 67	+0 73
21	0 70	0 75	0 76	+0 73	0 67	0 60	0 53	0 47	0 43	0 41	0 44	0 51	0 58	0 65	0 71
22	0 64	0 67	0 67	+0 62	0 56	0 48	0 42	0 37	0 33	0 35	0 40	0 47	0 53	0 60	0 67
23	0 55	0 57	0 54	+0 49	0 42	0 35	0 29	0 24	0 24	0 27	0 33	0 38	0 48	0 55	0 60
24	0 45	0 43	0 39	+0 33	0 26	0 20	+0 13	+0 12	0 13	0 17	0 22	0 31	0 40	0 46	0 50
25	+0 29	+0 28	+0 22	+0 16	+0 10	+0 02	−0 01	−0 02	+0 01	+0 05	+0 14	+0 24	+0 31	+0 36	+0 37
26	+0 15	+0 11	+0 06	−0 01	−0 09	−0 14	−0 16	−0 16	−0 13	−0 04	+0 06	0 13	0 20	0 23	0 23
27	0 02	−0 06	−0 12	−0 20	−0 26	−0 30	−0 31	−0 30	−0 22	−0 13	−0 04	+0 04	+0 08	+0 10	+0 08
28	−0 17	−0 22	−0 30	−0 37	−0 42	−0 46	−0 46	−0 39	−0 31	−0 22	−0 13	−0 07	−0 02	−0 03	−0 07
29	−0 31	−0 38	−0 46	−0 52	−0 57	−0 59	−0 54	−0 47	−0 39	−0 29	−0 22	−0 15	−0 13	−0 15	−0 20
30	−0 45	−0 52	−0 60	−0 66	−0 69	−0 66	−0 60	−0 54	−0 44	−0 36	−0 27	−0 23	−0 23	−0 26	−0 32

Table VIII, δv—Continued

Horizontal Argument, or Argument at top, D−g, or D−g+30   Vertical Argument, g

g	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″
0	−0 25	−0 35	−0 43	−0 48	−0 48	−0 46	−0 40	−0 31	−0 23	−0 14	−0 09	−0 07	−0 08	−0 12	−0 18	−0 26
1	−0 41	−0 51	−0 58	−0 60	−0 60	−0 56	−0 48	−0 40	−0 30	−0 25	−0 20	−0 20	−0 22	−0 28	−0 35	−0 42
2	−0 56	−0 64	−0 69	−0 71	−0 68	−0 62	−0 55	−0 45	−0 39	−0 33	−0 31	−0 32	−0 36	−0 42	−0 50	−0 57
3	−0 68	−0 71	−0 79	−0 78	−0 74	−0 67	−0 58	−0 51	−0 44	−0 40	−0 39	−0 42	−0 47	−0 55	−0 63	−0 70
4	−0 76	−0 82	−0 83	−0 81	−0 76	−0 67	−0 60	−0 52	−0 47	−0 45	−0 46	−0 50	−0 57	0 65	−0 73	−0 77
5	−0 81	−0 84	−0 84	−0 80	−0 74	−0 66	−0 58	−0 52	−0 48	−0 48	−0 50	−0 56	−0 64	−0 72	−0 78	−0 80
6	−0 80	−0 83	−0 81	−0 76	−0 69	−0 61	−0 51	−0 40	−0 47	−0 47	−0 52	−0 60	−0 68	−0 74	−0 78	−0 81
7	−0 77	−0 77	−0 74	−0 68	−0 60	−0 53	−0 47	−0 43	−0 42	−0 46	−0 52	−0 60	−0 67	−0 73	−0 77	−0 77
8	−0 69	−0 69	−0 64	−0 57	−0 50	−0 43	−0 39	−0 35	−0 37	−0 42	−0 50	−0 57	−0 63	−0 69	−0 71	−0 70
9	−0 60	−0 56	−0 51	−0 43	−0 37	−0 31	−0 26	−0 26	−0 30	−0 36	−0 43	−0 51	−0 58	−0 62	−0 62	−0 59
10	−0 46	−0 42	−0 35	−0 28	−0 22	−0 16	−0 14	−0 16	−0 21	−0 27	−0 35	−0 43	−0 49	−0 52	−0 51	−0 45
11	−0 30	−0 25	−0 19	−0 12	−0 05	−0 01	−0 02	−0 04	−0 10	−0 18	−0 27	−0 34	−0 39	−0 40	−0 36	−0 29
12	−0 13	−0 05	−0 01	+0 07	+0 12	+0 13	+0 12	+0 08	0 00	−0 09	−0 17	−0 23	−0 26	−0 25	−0 20	−0 13
13	+0 03	+0 10	+0 18	0 24	0 27	0 25	0 25	0 18	+0 10	+0 01	−0 06	−0 12	−0 13	−0 10	−0 01	+0 05
14	0 20	0 28	0 35	0 40	0 43	0 41	0 35	0 25	0 19	0 10	+0 03	0 00	+0 01	+0 05	+0 12	0 22
15	+0 37	+0 44	+0 51	+0 55	+0 55	+0 51	+0 44	+0 35	+0 26	+0 17	+0 13	+0 11	+0 13	+0 19	+0 28	+0 36
16	0 51	0 58	0 65	0 66	0 63	0 58	0 50	0 41	0 33	0 25	0 21	0 21	0 25	0 32	0 40	0 51
17	0 63	0 71	0 74	0 73	0 69	0 65	0 54	0 45	0 36	0 30	0 28	0 29	0 35	0 42	0 52	0 63
18	0 73	0 78	0 78	0 77	0 72	0 64	0 55	0 45	0 38	0 33	0 33	0 36	0 41	0 51	0 62	0 71
19	0 77	0 80	0 81	0 78	0 71	0 62	0 52	0 43	0 37	0 34	0 35	0 39	0 47	0 57	0 68	0 76
20	+0 77	+0 80	+0 79	+0 74	+0 66	+0 56	+0 46	+0 39	+0 34	+0 32	+0 31	+0 40	+0 50	+0 60	+0 70	+0 77
21	0 76	0 76	0 73	0 67	0 57	0 47	0 39	0 31	0 25	0 28	0 32	0 40	0 50	0 59	0 68	0 73
22	0 70	0 68	0 64	0 55	0 45	0 36	0 28	0 23	0 20	0 22	0 28	0 37	0 46	0 55	0 62	0 66
23	0 60	0 58	0 51	0 42	0 32	0 23	0 16	+0 11	0 11	0 15	0 22	0 30	0 40	0 48	0 55	0 57
24	0 40	0 43	0 35	0 26	+0 16	+0 08	+0 0[illegible]	−0 01	+0 01	+0 06	0 1[illegible]	0 22	0 31	0 38	0 43	0 44
25	+0 34	+0 27	+0 19	+0 0[illegible]	0 00	−0 07	−0 13	−0 14	−0 11	−0 05	+0 04	+0 12	+0 20	+0 27	+0 30	+0 29
26	0 18	+0 11	+0 01	−0 08	−0 17	−0 24	−0 27	−0 26	−0 23	−0 15	−0 07	+0 02	+0 10	+0 14	+0 1[illegible]	+0 12
27	+0 02	−0 06	−0 16	−0 25	−0 34	−0 39	−0 41	−0 39	−0 33	−0 26	−0 16	−0 08	−0 02	0 00	−0 01	−0 05
28	−0 14	−0 23	−0 32	−0 42	−0 49	−0 53	−0 54	−0 50	−0 43	−0 37	−0 25	−0 19	−0 14	−0 14	−0 17	−0 20
29	−0 28	−0 37	−0 48	−0 57	−0 63	−0 65	−0 64	−0 58	−0 50	−0 41	−0 34	−0 28	−0 26	−0 27	−0 29	−0 35
30	−0 41	−0 51	−0 62	−0 69	−0 73	−0 74	−0 70	−0 64	−0 55	−0 47	−0 41	−0 37	−0 37	−0 37	−0 42	−0 49

Table IX

Argument, g Factor to be multiplied by n δz

g	F
0	+ 0 118
1	0 114
2	0 103
3	0 086
4	0 065
5	+ 0 040
6	+ 0 015
7	− 0 009
8	− 0 034
9	− 0 054
10	− 0 072
11	− 0 086
12	− 0 096
13	− 0 101
14	− 0 103
15	− 0 099
16	− 0 092
17	− 0 080
18	− 0 065
19	− 0 046
20	− 0 024
21	+ 0 001
22	0 026
23	0 051
24	0 075
25	+ 0 094
26	0 109
27	0 116
28	0 117
29	0 110
30	+ 0 096

Table X

Argument, u Factors for correction of latitude and reduction to ecliptic longitude

u	$(F\,l)$	$(F\,\beta)$	$\delta\beta_1$
			″
0	− 0 004	+ 0 090	0 00
1	− 0 004	0 088	− 0 03
2	− 0 003	0 081	− 0 07
3	− 0 001	0 069	− 0 10
4	+ 0 001	0 054	− 0 12
5	+ 0 003	+ 0 036	− 0 14
6	0 004	+ 0 017	− 0 15
7	0 004	− 0 004	− 0 15
8	0 004	− 0 024	− 0 14
9	+ 0 002	− 0 044	− 0 13
10	0 000	− 0 060	− 0 11
11	− 0 001	− 0 074	− 0 08
12	− 0 003	− 0 084	− 0 05
13	− 0 004	− 0 089	− 0 02
14	− 0 004	− 0 089	+ 0 01
15	− 0 003	− 0 085	+ 0 05
16	− 0 002	− 0 076	0 08
17	0 000	− 0 064	0 11
18	+ 0 002	− 0 047	0 13
19	0 003	− 0 028	0 14
20	+ 0 004	− 0 008	+ 0 15
21	0 004	+ 0 012	0 15
22	0 003	0 032	0 14
23	+ 0 001	0 050	0 12
24	0 000	0 066	0 10
25	− 0 002	+ 0 078	+ 0 07
26	− 0 004	0 086	0 04
27	− 0 004	0 090	+ 0 01
28	− 0 004	0 089	− 0 03
29	− 0 003	0 082	− 0 06
30	− 0 001	+ 0 072	− 0 09

TABLE XI

Factors for converting small changes of longitude and latitude into changes of right ascension and declination
Arguments, ☽'s longitude and latitude

FORMULAE $\delta\alpha = \delta v + (v\ \alpha)\,\delta v + (\beta\ \alpha)\,\delta\beta$,
$\delta\delta = \delta\beta + (v\ \delta)\,\delta v + (\beta\ \delta)\,\delta\beta$

☽'s long	$(v\ \alpha)$			$(\beta\ \alpha)$			$(v\ \delta)$	$(\beta\ \delta)$	
	$\beta = -5°$	0°	+5°	$\beta = -5°$	0°	+5°			
°									°
270	+ 133 +	+ 090 +	+ 050 +	000	000	000	000	000	270
275	131	089	049	− 043 +	− 041 +	− 040 +	+ 038 −	− 001 −	265
280	126 +	084 +	045 +	− 085 +	− 081 +	− 079 +	075 −	− 003 −	260
285	+ 117	+ 076	+ 039	− 126	− 121	− 117	+ 111 −	− 006 −	255
290	105	066	030	− 165	− 158	− 154	147 −	− 011 −	250
295	091 +	055 +	021 +	− 202 +	− 193 +	− 188 +	180 −	− 016 −	245
300	+ 074	+ 041	+ 009 +	− 235	− 226	− 220	+ 212 −	− 023 −	240
305	057	027	− 003 −	− 265	− 256	− 250	242 −	− 030 −	235
310	039 +	+ 011 +	− 016 −	− 292 +	− 282 +	− 277 +	269 −	− 037 −	230
315	+ 021	− 004 −	− 028 −	− 316	− 306	− 301	+ 293 −	− 044 −	225
320	+ 003 +	− 018 −	− 040 −	− 336	− 326	− 322	315 −	− 051 −	220
325	− 014 −	− 032 −	− 052 −	− 353 +	− 344 +	− 340 +	335 −	− 058 −	215
330	− 030 −	− 045 −	− 061 −	− 368	− 359	− 356	+ 352 −	− 064 −	210
335	− 044 −	− 056 −	− 070 −	− 379	− 371	− 369	366 −	− 069 −	205
340	− 056 −	− 065 −	− 077 −	− 388 +	− 381 +	− 379 +	378 −	− 074 −	200
345	− 066 −	− 073 −	− 082 −	− 395	− 388	− 388	+ 386 −	− 078 −	195
350	− 074 −	− 078 −	− 085 −	− 399	− 394	− 394	393 −	− 080 −	190
355	− 080 −	− 081 −	085 −	− 401 +	− 397 +	− 399 +	397 −	− 082 −	185
0	− 084 −	− 083 −	− 084 −	− 401	− 398	− 401	+ 398 −	− 082 −	180
5	− 085 −	− 081 −	− 080 −	− 399	− 397	− 401	397 −	− 082 −	175
10	− 085 −	− 078 −	− 074 −	− 394 +	− 394 +	− 399 +	393 −	− 080 −	170
15	− 082 −	− 073 −	− 066 −	− 388	− 388	− 395	+ 386 −	− 078 −	165
20	− 077 −	− 065 −	− 056 −	− 379	− 381	− 388	378 −	− 074 −	160
25	− 070 −	− 056 −	− 044 −	− 369 +	− 371 +	− 379 +	366 −	− 069 −	155
30	− 061 −	− 045 −	− 030 −	− 356	− 359	− 368	+ 352 −	− 064 −	150
35	− 052 −	− 032 −	− 014 −	− 340	− 344	− 353	335 −	− 058 −	145
40	− 040 −	− 018 −	+ 003 +	− 322 +	− 326 +	− 336 +	315 −	− 051 −	140
45	− 028 −	− 004 −	+ 021	− 301	− 306	− 316	+ 293 −	− 044 −	135
50	− 016 −	+ 011 +	039	− 277	− 282	− 292	269 −	− 037 −	130
55	− 003 −	027 +	057 +	− 250 +	− 256 +	− 265 +	242 −	− 030 −	125
60	+ 009 +	+ 041	+ 074	− 220	− 226	− 235	+ 212 −	− 023 −	120
65	021	055	091	− 188	− 193	− 202	180 −	− 016 −	115
70	030 +	066 +	105 +	− 154 +	− 158 +	− 165 +	147 −	− 011 −	110
75	+ 039	+ 076	+ 117	− 117	− 121	− 126	+ 111 −	− 006 −	105
80	045	084	126	− 079	− 081	− 085	075 −	− 003 −	100
85	049 +	089 +	131 +	− 040 +	− 041 +	− 043 +	+ 038 −	− 001 −	95
90	+ 050 +	+ 090 +	+ 133 +	000	000	000	000	000	90
	$\beta = -5°$	0°	+5°	$\beta = -5°$	0°	+5°			☽'s long

INSTRUCTIONS

FOR

OBSERVING THE TRANSIT OF VENUS,

DECEMBER 6, 1882,

PREPARED BY

THE COMMISSION AUTHORIZED BY CONGRESS,

AND

PRINTED FOR THE USE OF THE OBSERVING PARTIES BY AUTHORITY OF THE HON. SECRETARY OF THE NAVY.

WASHINGTON
GOVERNMENT PRINTING OFFICE.
1882.

[GENERAL ORDER.]

NAVY DEPARTMENT, *Washington, August* 11, 1882.

The parties organized by the Secretary of the Navy to observe the Transit of Venus in December, 1882, under the authority of an act of Congress approved August 7, 1882, are organizations invested with naval character, and subject to naval rules, regulations, and discipline. The command of each party is assigned to the Chief Astronomer, to whose authority all others will be obedient.

The following is the order of rank and authority in each party, viz:

1. Chief Astronomer.
2. Assistant Astronomer.
3. Chief Photographer.
4. Assistant Photographer.

This order will be respected and followed as the order of rank and succession in each party under all circumstances, including the contingencies growing out of any separation of the party, or the happening of any vacancy.

The party destined for the coast of Patagonia, about to embark in the U. S. steamer Brooklyn, will be personally subject to the authority of the commanding officer while on board that ship, and to its discipline, police authorities, and regulations.

The gentlemen engaged in this interesting and important service will readily and fully understand that discipline, harmony, and co-operation are essential to its satisfactory performance as well as to their own safety and convenience, and that for this reason they have been required to assume the obligations, and are subjected to the rules of the naval service.

WM. E. CHANDLER,
Secretary of the Navy.

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NAVY DEPARTMENT, *Washington, August* 11, 1882

The following instructions, prepared by the Commission on the Transit of Venus, are issued, under the authority of this Department, for the guidance and government of the parties charged with the duty of making the observations of the Transit Every person engaged in the scientific operations is enjoined to study them carefully and to conform to them strictly

WM E CHANDLER,
Secretary of the Navy

INSTRUCTIONS

FOR

OBSERVING THE TRANSIT OF VENUS.

DECEMBER 6, 1882.

I.—INTRODUCTORY NOTE.

The following instructions were prepared primarily for the use of the parties organized by the U. S. Transit of Venus Commission. But as the transit will be visible in this country, they have also been adapted to the use of amateur observers who desire to be made acquainted with the methods by which they may make observations of value.

II.—SELECTION OF STATION.

In choosing ground for a station, it is most important to obtain good foundations for the instruments. Gravel is best, but a sandy soil is unobjectionable, if the sand is not dr y.

A nearly level spot, measuring at least sixty feet from north to south, is necessary for setting up the transit instrument and photoheliograph. The floor of the photographic house must be eight and a half inches lower than that of the transit house, and it will be advantageous if the natural slope of the ground gives that amount of fall; but if it does not, one or other of the houses must be elevated to obtain the proper difference of level between them. For the sake of dryness, the sills of the houses should be raised three or four inches above the surface of the soil. In many situations this can be convenietly done by driving stakes about four inches square into the ground, setting the houses upon them, and nailing them fast. In other places it may be necessary to use small piers of stone or brick.

Shelter from the prevailing winds is to be sought for; but great care must be taken that the sun be visible from the point occupied by the photographic objective, and by the equatorial telescope, during the whole time of the transit, and, indeed, a little longer. For this and other purposes, the observers must make sure of the exact local times of all the contacts.

After the site for the station is selected, a survey of the surrounding region must be made to determine the location of the station relatively to neighboring permanent objects. If a large scale map of the region can be obtained, the position of the station may be marked upon it; but if no such map is available, one must be made.

A carefully-written description of the position of the station must also accompany the map The object of these records is to provide means for relocating the station within one or two hundred yards at any future time

III —LAYING OFF GROUND, AND SETTING UP INSTRUMENTS.

The approximate position of the meridian-line, on which are to be placed the three piers for supporting the transit, the photographic objective, and the plate-holder, being determined by compass or otherwise, the next two operations, which may be carried on together, will be the laying down of an exact meridian-line and the erection of a pier for the transit

As soon as the position of the transit pier is decided, a point should be selected about seven feet east or west of its center, from which to lay off a meridian-line by the theodolite The latter instrument being placed over this point, a stake should be driven nearly south or north of it, and at a distance of not less than one hundred feet, and then the azimuth of the line joining the instrument and the stake should be found from observations of the sun or stars made on both sides of the meridian From this line of known azimuth a true meridian-line should next be laid down, and that can be done either with the theodolite, or by setting off a proper offset from the stake Perhaps the latter is the better method

The foundation of the transit pier should be three or four feet below the surface of the ground, and, in northern stations, below the reach of frost, if practicable The last earth at the bottom of the excavation should be carefully removed with a spade in horizontal slices, so as to produce a clean, hard, surface for the footing course to rest upon It is desirable to prevent the surface earth pressing against the sides of the pier, and for that reason a vacant space one or two feet deep should be left around it—a curb being used to keep back the earth, if necessary The sides of the pier must face the cardinal points accurately Above ground, it should measure twenty-one inches from north to south, and twenty-five inches from east to west, below, it may be from three to three and a half feet square The cap-stone must be twenty-four by twenty-eight inches, and three inches thick The transit house must be built around the pier after the latter is erected, the floor of the house being placed twenty-nine inches below the top of the pier

In the northern hemisphere the photographic house will be south, and in the southern hemisphere north, of the transit house The positions and depths of the holes for the iron piers of the photoheliograph can be readily determined by means of the meridian line and the numbers in TABLE I The holes may be dug at any time, but the piers cannot be set until the transit instrument has been brought into the meridian and its errors made as small as possible The lower ends of the piers have thin edges, and to prevent them settling into the ground they should be placed upon large flat stones in the bottoms of the holes The conditions to be fulfilled in erecting the piers are as follows

1 The flanged ends of the piers must be uppermost

2. The larger of the two piers carries the photographic objective, and goes nearest the transit house Its center must be about fourteen feet from the center of the transit pier.

3. The distance between the centers of the two piers must be as stated in TABLE I.

4. The height of the upper surface of the flange of each pier, or, in other words, the height of its top, must be as stated in TABLE I.

5. The centers of the two piers must be in the plane of the true meridian passing through the transit instrument.

6. The piers must stand perfectly plumb.

7. The screw-holes in the flanges of the piers must be so placed that in each pier the hole nearest the transit house is in the plane of the meridian—that is, in the plane passing through the centers of the piers. The aperture in the side of the plate-holder pier must also be in the meridian, but must face away from the transit house.

The simultaneous fulfillment of so many conditions is troublesome. A convenient way of proceeding will be to tack a lath across the slit in the transit house at a point four feet above the floor, and from it to stretch a cord, perhaps sixty feet long, in the meridian of the transit instrument and horizontally. A strip of wood must also be jammed into the top of each iron pier in such a way that a five-penny nail can be driven into it, and be left projecting an inch, to mark the center of the pier. Then, by sticking pins through the line at the points beneath which the centers of the piers must be located, the latter can be brought very approximately into position. In making the final adjustments, an engineer's level and a theodolite will be required. The engineer's level should be set up about sixty feet from the transit pier, and in such a position that the latter can be seen through the door of the transit house. The theodolite must be mounted behind the transit house, and in the meridian of, but a little higher than, the transit instrument, so that the tops of the iron piers can be seen over the latter. By moving the transit instrument in altitude only, and the theodolite in both altitude and azimuth, the two instruments must be pointed on each other, and the image of the central wires of the transit must be brought into coincidence with the central wires of the theodolite, and then the line of collimation of the latter will describe a plane parallel to the meridian of the former. This suffices for our purpose; but if it is desired to bring the meridians of the two instruments into absolute coincidence, it can be done by shifting the theodolite towards the east or west until the images of the two objectives given by the theodolite eye-piece are seen to be concentric. For that purpose a magnifier is necessary, and care must be taken that the coincidence of wires of the two instruments is left perfect. Thus adjusted, the theodolite will show with great precision when the centers of the piers are in the meridian of the transit instrument; and the engineer's level will show equally exactly when their tops are at the proper height.

Instead of using a theodolite to place the piers in the meridian, the transit instrument itself may be employed, if its objective is covered by a cap having a diametral slit one-fifth of an inch wide. This slit must be placed truly vertical, and then it will be possible to obtain somewhat indistinct vision of any vertical line situated in the meridian of the instrument and having a diameter not less than the width of the slit. If the centers of the piers are marked by twenty-penny nails, instead of five-penny ones, they can be seen through the transit. To guard against error, after a pier has been apparently brought into the meridian, the cap should be twisted through

half a revolution, and if that produces any change in the position of the image in the transit, the mean of the two positions will be the true one, and the center of the pier must be adjusted accordingly

The pier for the photographic objective should first be brought accurately into position, and afterwards the same thing should be done for the plate-holder pier In adjusting the distance between the two piers a steel tape-line must be used, not a linen one Nails answer well as wedges for making small changes in the heights of the piers When all is ready, the piers must be finally fixed by filling the holes around them with masonry laid in cement, or with cement concrete—earth does not give sufficient firmness, and in doing this special care must be taken that they are not accidentally shifted from their true positions Their interiors should also be filled with concrete to the level of the ground

The photographic house must be built around the plate-holder pier, the floor of the house being placed exactly three feet ten and three-quarter inches below the upper surface of the flange of the pier

The cast-iron plates which support the photographic objective and plate-holder are next to be fixed in position The coffin-shaped plate is to be bolted to the larger pier, its long end being turned toward the photographic house, and at least one washer being placed around each bolt, between the plate and the pier, so as to give the former a solid bearing The coffin plates are not all alike The older ones have pipes at the end, to receive the prongs of the piece which supports the objective These pipes must be turned downward The newer ones have a planed seat, upon which the bottom of the support for the objective is bolted This seat must face upward When the objective-carrier is first mounted upon the coffin plate, the screw, or screws, which hold it should be only lightly set up, but afterwards, when all has been found right, they must be firmly turned home, and the pipes, if there are any, may be filled with plaster or cement The round cast-iron disc which carries the plate-holder is to be secured to the pier in the photographic house by the proper supporting and binding screws, care being taken that it is turned in the right direction

To mount the plate-holder, the brass cross must be screwed to the cast-iron disc on top of the plate-holder pier, the tube of the cross passing down through the hole in the disc The vertical axis of the plate-holder is to be set in this tube, and when the plate-holder is rotated its ends should just graze the raised ends of the cross below it Milled-headed screws, passing through slots in pieces at the ends of the arms of the cross, are provided for fixing the plate-holder in position, with the reticule plate at right angles to the optical axis of the objective

The structure for carrying the iron measuring-rod and the tube of the photoheliograph should next be erected, and the measuring-rod, but not the tube, must be mounted. The last frame of the structure should be two feet distant from the nearest edge of the coffin-plate, and the position of the measuring-rod must be parallel to, but nine inches above, the optical axis of the photographic objective For further details, consult section XI.

IV.—MEMORANDA RESPECTING THE PHOTOHELIOGRAPH

For convenience of reference the measurements required in erecting the horizontal photoheliograph are here recapitulated.

The houses may be set up on stakes 4″ square, driven firmly into the ground. The upper surfaces of the floors should be 7″ or 8″ above the surface of the ground. The distance from the upper surfaces of the floors to the bottoms of the sills is about 3½″.

Size of transit pier below ground, 3′ or 3′ 6″ square, above ground, 21″x25″. Capstone for ditto, 24″x28″x3″. Top of pier above floor, 29″. Eye-piece of transit above floor, 3′ 9½″.

The iron piers for the photoheliograph are 8′ 0″ long. The larger of the two carries the objective.

Distance from center of transit pier to center of pier for photographic objective, 14′ 0″.

Photographic house. Floor 8½″ lower than floor of transit house. Distance from inner side of wall of house to center of plate-holder pier, 12″. The top of this pier is 9¼″ above top of transit pier, and 3′ 10¾″ above floor of photographic house. Center of plate-holder above floor, 4′ 6″.

In TABLE I, the quantities on each line refer to a single photoheliograph. The number of the objective is given in column A. The distance between the back surface of the objective and the sensitive surface of the photographic plate is given in column B, it being assumed that the thickness of the reticule plate is 0.25 of an inch. The distance of the back surface of the objective from its second principal point is given in column C. The distance between the centers of the piers which carry the objective and plate-holder is given in column D. The height of the top of the objective pier above the top of the transit pier is given in column E. The number of the measuring-rod to be used with each objective is given in column F. The length of each measuring-rod, at 62° Fahrenheit, is given in column G.

TABLE I.

A.	B	C	D	E	F	G
	Inches	Inch	Inches	Inches		Inches
1	462.51	0.711	477.6	10.4	VIII	451.491
2	465.08	.682			VII	453.498
3	462.27	.674	476.9	9.4	II	450.437
4	463.33	.718	478.8	10.4	VI	451.946
5	464.79	.538	479.9	10.4	III	453.488
6	472.90	.708	488.5	10.4	I	461.425
7	461.20	.644	476.4	10.4	V	449.485
8	461.30	0.708	476.0	9.4	IV	450.357

The expansion of the measuring-rods may be taken as 0.0000070 of their length for one degree Fahrenheit.

V.—ADJUSTMENTS OF THE PHOTOHELIOGRAPH.

The photoheliograph must fulfill the following conditions

1 The sensitive surface of the photographic plate must be at the focus of the objective

2 The line joining the optical center of the object-glass and the cross-lines in the middle of the reticule plate must be in the true meridian, within a fraction of a minute of arc

3 The same line must be horizontal, within the same limits

4 The optical axis of the objective must be directed toward the center of the reticule plate

5 The reticule plate must be perpendicular to the line joining its center and the center of the objective

6 To let the plumb line hang freely, the sides of the plate-holder must be vertical, and that will be attained by making its top level

These adjustments are made as follows

1 By the aid of the measuring-rod, set the coffin-plate so that the distance between the back surface of the objective and the sensitive surface of the photographic plate is as stated in column B of TABLE I To guard against errors, after the adjustments 1 to 4 have been made, remove the brass plate-holder and use the wooden one to take a number of photographs of the sun at different distances inside and outside the focus. The points at which the small spots on the sun begin to disappear when the plate is too far in, and again when it is too far out, must be noted The mean of the two positions is the true focus If, after repeated trials, it differs more than one-quarter of an inch from the point found by measurement, the position of the objective must be changed accordingly

2 When the photographic objective is in position, point the telescope of the transit instrument at it, and set a bull's-eye lantern behind the center of the reticule-plate The lines upon the latter will then be visible through the transit, but not very distinctly, because the photographic focus of the photoheliograph differs considerably from its visual focus If there is any difficulty in seeing and identifying the intersection of the central lines, gum a small triangular bit of paper upon the reticule-plate with one of its angles at the point in question; or rule a little cross with ink, making its lines not more than half an inch long, and taking care that they coincide exactly with the etched lines of the plate If the azimuth and collimation of the transit are quite right, its middle vertical wire should be on the middle vertical line of the reticule-plate If it is not found so, the error must be corrected by moving the brass cross which carries the plate-holder

3 Point the transit so that its middle horizontal wire accurately coincides with the image of the middle horizontal line of the reticule-plate, and clamp it firmly in that position Then set up a carefully adjusted engineer's level between the transit house and the photographic objective, point it into the latter, and bring its horizontal wire into accurate coincidence with the image of the middle horizontal line of the reticule-plate Read the bubble of the level, and if it is within six or eight divisions of the

middle of its scale the height of the plate-holder is probably satisfactory. To make sure of this, point the level at the transit, bring its horizontal wire into exact coincidence with the middle horizontal wire of the latter, and again read its bubble. Half the distance traveled by the bubble between the two readings will be the error of level of the center of the reticule-plate.

Probably at the first trial the bubble will run all the way to one end or the other of its tube. If, at each pointing, it runs to the end nearest the photographic house, the reticule-plate is too high, but if to the end nearest the transit house, the reticule-plate is too low. In either case the error must be corrected by changing the elevation of the plate-holder, or by changing the elevation of the coffin-plate, or by both. The height of the plate-holder is controlled by the adjusting-screws of the iron disk supporting it, and the altitude of the coffin-plate may be modified by increasing or diminishing the number of washers under it.

If instead of an engineer's level a level of precision is employed, its telescope must be pointed at the center of the reticule-plate, and its bubble must be read, reversed, and read again. Let the difference of these two readings be A. Then the telescope must be rotated about its optical axis through half a revolution, once more pointed at the center of the reticule-plate, and, as before, the bubble must be read, reversed, and read again. Calling the difference of this last pair of readings B, the error of level of the center of the reticule-plate will be $\frac{1}{4}(A+B)$. The object in rotating the telescope through half a revolution is to eliminate its collimation-error.

4. After the plate-holder is fixed in its true position, adjust the objective by its three supporting-screws so that if a candle in the photographic house is held in the line passing through the centers of the objective and reticule-plate, its three reflections from the objective will also lie in the same line. Instead of a candle, it is sometimes convenient to use a reflector consisting of a card-board disc two or three inches in diameter, with a hole one-quarter of an inch in diameter through its center. In that case, the three images of the hole reflected from the objective must lie in the line in question.

5. Adjust the reticule-plate so that if a candle in the photographic house is held as far as possible from it, and in the line passing through its center and the center of the objective, the reflection of the candle from the reticule-plate will also lie in the same line. Here, again, a card-board reflector may be used instead of a candle. The adjustment is made in altitude by the screws supporting the iron disc which carries the plate-holder, and in azimuth by turning the latter on its vertical axis. When all is right, the screws confining the plate-holder must be firmly turned home.

If this adjustment is correctly made the surfaces of the reticule-plate will be vertical. This may be independently tested in two ways.

(a) Set the engineer's level outside the house, at the height of, and near the central line joining the objective and plate-holder, in either direction from the latter, and at such distance from it, not less than 15 or 20 feet, as will be favorable for the observation. Point the level at the plate-holder, turn the latter so that a horizontal line upon the surface of the reticule shall be perpendicular to the line from the level, and adjust the focus of the level so that its objective can be seen by reflection from the

reticule-plate If the plate is vertical, and the level properly adjusted, the horizontal wire of the level will bisect the reflected image of the objective The latter should return to its position when the plate-holder is turned half-way round on its axis, so that the reflection takes place from the other surface

(*b*) Adjust the base of the plate-holder so that the bubble of a level set upon its top shall not vary greatly in position when the holder is turned on its axis

If these two tests cannot be both satisfied within one or two minutes of arc by the same adjustment, the chief should endeavor to ascertain what is wrong, though it may not be advisable for him to try to correct it

6 The verticality of the sides of the plate-holder is controlled by the supporting screws of the iron disk which carries it An ordinary carpenter's level suffices to show when the adjustment is correct

The adjustments from 1 to 6 are necessarily made consecutively, and they must be gone over a second time to make sure that the later ones have not disturbed those first established

VI —THE HELIOSTAT.

As the heliostat will presently be needed, it should now be set in front of the objective, upon the coffin-shaped plate, and the three adjustments which it requires should be made They are as follows 1 Its main axis must be brought into the plane of the meridian, 2 Its main axis must be set at that inclination which will keep the sun's image most nearly at a constant height upon the reticule plate; 3. The driving-clock must be arranged to rotate the main axis at the proper speed Extreme accuracy in these adjustments is superfluous, because, in order to obtain freedom from vibration, a form of heliostat has been adopted which can throw the sun's rays only approximately in a constant direction A convenient way of proceeding, and one which will probably give as good results as any, will be to make the first adjustment by estimation, using a ruler laid against the cube of the main axis to aid the judgment; and then to effect the second and third adjustments by trial

If more exactness is desired, the first adjustment may be made by setting the mirror at right angles to the main axis of the heliostat, and then setting the latter so that the mirror is also at right angles to the axis of the photographic telescope The setting of the mirror is accomplished when the direction of a ray reflected from its first surface is not affected by rotating the main axis, but in applying this test care must be taken not to mistake the ray reflected from the second surface for that from the first Then the main axis is brought into the plane of the meridian, and made horizontal, by setting the heliostat so that it will reflect back upon itself the light from a candle held at the center of the reticule-plate The proper inclination for the main axis may be computed, and the axis can be set by means of a clinometer; but the rating of the driving-clock can only be effected by trial

Whenever the azimuth and level of the center of the reticule-plate are determined, the heliostat will have to be removed from the coffin plate; and to save trouble in returning it to its proper place, the points where its feet rest should be marked.

The driving-clock is provided with three pulleys, whose time of revolution may be varied from 37.8 seconds to 42.3 seconds by raising or lowering the pendulum bob. The screw of the heliostat carries two wheels, either of which can be connected by a leather band to any one of the driving-clock pulleys; and by making suitable combinations, the screw can be driven at any desired speed between the limits 58.8 seconds and 115.0 seconds per revolution. This suffices for all localities. Sometimes the pendulum of the driving-clock takes a wabbling motion, moving in an ellipse instead of a circle. When this happens it is generally occasioned by friction at the point of suspension, and a little oil will remedy the difficulty.

VII.—THE EXPOSING SLIDE.

The frame carrying the exposing slide must be screwed to the inner surface of the wall of the photographic house in such a position that the line joining the centers of the objective and reticule-plate passes through the center of the opening in the frame. Upon each end of that surface of the slide which is nearest the objective a target is painted, and whenever the slide is brought into contact with the pieces which limit its motion, one other of these targets covers the aperture in the frame, and is visible to a person standing at the objective. If the image of the sun given by the latter is then centered upon the target, it is intended that it shall also be found centered upon the reticule-plate when the slide is moved across the opening in the frame. To secure this result, the adjustments of the photoheliograph should be completed before the frame is put up, and special pains should be taken to fix it exactly in its right position.

By means of the six milled-headed screws upon the slide, the brass plates can be set so as to give any desired width of slit; but in doing this care must be taken to keep the center of the slit coincident with the center of the opening in the slide, because the automatic key for recording the instant of exposure upon the chronograph is arranged to break when the centers of the openings in the slit and its frame coincide with each other. It is sometimes desirable to see the entire image of the sun upon the reticule-plate, and the slit is arranged to open wide enough for that.

VIII.—THE TUBE.

Experience has shown that for a photoheliograph of thirty-eight and a half feet focus twelve feet of tube is sufficient. For convenience of transportation, that furnished to the parties is slightly conical, and in two-feet lengths packed inside each other. In mounting the tube, four points require attention, namely: 1. The largest end of the tube should pass snugly through the wall of the photographic house and rest against the back of the exposing-slide frame. 2. The centers of the openings in all the diaphragms should be in the straight line joining the centers of the objective and reticule-plate. 3. From the reticule-plate nothing but black surfaces should be visible. The presence of white light risks fogging the photographic plates, and therefore this condition is imperative. To fulfill it, that one of the frames carrying the measuring-rod which is nearest the objective must have its upper part boarded over so as to shut out extraneous light from the tube. Both surfaces of this screen should

be colored dead black, and a hole must be cut through it just large enough to permit the free passage of rays from all parts of the objective 4 It is absolutely necessary that both the tube and the measuring-rod be thoroughly protected from the sun's rays As there must be an air-space of from six to twelve inches around the tube, the roof or awning covering it should have a depth not less than three and one-quarter feet Beyond the termination of the tube, the covering for the measuring-rod may be formed of three boards, each eight inches wide, put together so as to form a kind of inverted trough

IX —THE PLATE-HOLDER PLUMB LINE

Both the chief of party and the chief photographer must give special attention to the plate-holder plumb-line The wire employed is of gilded brass, having a length of about three feet and a diameter of 0 0025 of an inch It must be without any bends or kinks, must pass perfectly freely through the axis of the plate-holder, and must be loaded with one-fifth of its breaking weight To secure steadiness, the weight must hang in a vessel of water within the pier, care being taken that the vessel has sufficient size and is so placed as to avoid any risk of the weight resting against its bottom or sides The upper extremity of the plumb-line is wedged into a brass piece which fits into a socket in the top of the plate-holder, and is provided with an arm for rotating the wire through half a revolution, so as to eliminate the effect of any undetected kinks or bends Aside from this motion of rotation, the plumb-line should be disturbed as little as possible

The chief of party must satisfy himself from time to time that the plumb-line does not come into contact with anything in its passage through the opening in the axis of the plate-holder One way of testing this will be to let the bob swing through a minute arc, and see that the swinging motion of the suspending wire across the face of the reticule plate, as viewed with a magnifying glass, is perfectly regular

X —BATTERIES AND ELECTRICAL CONNECTIONS

The batteries furnished to the parties are of the Daniell's gravity form To set up a cell, unfold the copper element, place it, together with about two pounds of coarsely-powdered copper sulphate, in the bottom of the glass jar, and lead the insulated copper terminal out over the top of the jar If the cell is desired for immediate use, fill it to within one and a half inches of the top with water in which a little zinc sulphate has been dissolved, and suspend the zinc element in this solution by hooking it to the top of the jar If the cell is not required immediately, it is better to fill it with pure water, and then to connect the copper terminal with the zinc. In a few hours it will be in good working order When using these cells care must be taken that the zinc sulphate solution does not become too strong, that it covers the zinc element, and that there are always some copper sulphate crystals in the bottom of the jar

No stronger current than that from a single Daniell's cell should ever be passed through a break-circuit chronometer; and as that will not usually suffice for working a chronograph, the chronometer must be joined up in circuit with one cell of battery, a single-point switch, and a repeater of about six ohms resistance The break-circuit

points in the chronometer are very delicate, and to diminish the spark at them the lid of the chronometer box contains a condenser which should be included in the circuit The chronograph must be joined up in a second circuit, passing through the points of the repeater, and including the observing key at the transit instrument, the automatic key of the exposing slide in the photographic house, and such a number of battery cells as may be necessary Thus the chronometer circuit will control the chronograph circuit, and by opening the switch in the former, both circuits will be opened It will be well to arrange the wires in such a way that the automatic key in the photographic house can be cut out when not in use As a rule, only break-circuit signals will be used, but the observing keys furnished to the parties have a screw at the back, by shifting which they can be converted into make-circuit keys if desired

XI—THE MEASURING-ROD

The most convenient way of supporting the measuring-rod will be to pass it through holes one and one-quarter inches in diameter in boards nailed across the tops of A-shaped frames placed not more than six feet apart Whatever mode of support is adopted, special care must be taken to see that the rod lies perfectly straight, and that it is parallel to the optical axis of the photographic telescope

The rod is of wrought-iron gas-pipe, eight-tenths of an inch diameter, in sections five feet long, with Arabic numbers at the joints to show how they go together The Roman numbers in the middle of each section are the number of the rod The ends of each section must be carefully cleaned with an oily rag before they are united, and in screwing them together their shoulders must be made to meet snugly Marks at each joint show approximately the point of stopping If these marks fail to come together, there is dirt in the joint, but if they pass each other slightly, it only indicates that the screw is worn, which is of no consequence, because the length of the rod depends upon the condition of the shoulders and not upon that of the screws To avoid straining the rod in putting it together, the first two sections should be united and passed on to the supporting frames nearest the objective, then the third section should be added, and the rod should be pushed nearer the photographic house, and so on till it is all together, and in place The end inside the photographic house should be at the same distance from the front surface of the reticule plate that the end outside is from the back surface of the objective After being mounted, the rod should not again be disturbed till the party is about to leave the station Its outer extremity should be slightly greased to prevent it from rusting, and must be protected from the weather by the tin cover furnished for that purpose

The measuring-rod is used in connection with the jaw-micrometer for determining the interval between the back surface of the objective and the front surface of the reticule plate To do this, three thermometers must first be placed upon the rod to ascertain its temperature, one being at its center and one near each end. Then a plumb-line, consisting of a brass wire 0 0032 of an inch in diameter, must be hung over the outer end of the rod, the bob of the line hanging in a vessel of water, and being protected from wind by complete immersion in the fluid Care must be taken that the wire bends sharply over the end of the rod, and is in actual contact with its

front surface When the plumb-line has come to rest, press the arms of the jaw-micrometer against the margin of the object-glass, taking great care to hold the micrometer horizontally, and by the two rack motions bring the end of the central arm gently into contact with the back surface of the objective, and the jaws into such a position that the plumb-line is between them and in the line joining the centers of the pin-holes In adjusting the position of the pin-holes relatively to the plumb-line, it will be advantageous to use a magnifier of low power When all the adjustments are correct, read and record the vernier of the micrometer After making several such measurements at the objective, pass to the plate-holder and make similar measurements from the front surface of the reticule plate Each measure should be repeated a number of times by different observers, and each separate result should be recorded with the name of the observer, the temperature of the rod, and any other necessary particulars

Should the rod be too long, file a vertical notch on each side near one end, loop the plumb-line, and let it hang in these notches

The distance from the front surface of the reticule plate to the position of the collodion film must also be measured as accurately as possible

XII —INSTRUMENTAL ERRORS

When the adjustments of the photoheliograph designated 1, 4, 5, and 6, in section V, have once been made, it is expected they will remain sufficiently exact, unless purposely disturbed If, by any chance, either 4 or 5 is found wrong, it must be corrected. The errors of 1, 2, and 3 must be determined at least twice a week, but unless they become large they need not be corrected

To find the error of the first adjustment, the distance from the back surface of the objective to the front surface of the reticule plate must be measured with the greatest care at times when the temperature is not varying rapidly Directions for doing this are given in the preceding section

The error of the second adjustment must be found as follows. Every evening before beginning work with the transit instrument, place its eye-piece at the same side of the stand as the azimuthal adjusting screws, and by means of the latter, bring the middle vertical wire of the transit into exact coincidence with the image of the middle vertical line of the reticule plate After this, the azimuthal adjustment must remain undisturbed during the night's work Then make a set of time observations, that is, observe two azimuth stars above the pole, two below the pole, and six or eight time stars, one half the observations of each class being made with clamp east and the other half with clamp west At the close of the night's work again bring the eye-piece to the same side of the stand with the azimuthal screws, and examine the position of the middle wire relatively to the image of the middle vertical line of the reticule plate If there is any deviation, its amount must be estimated and recorded, but the azimuth of the transit must not be disturbed

Upon reducing the time observations the azimuth and collimation constants of the transit will become known Let them be a and c, and let the azimuth of the center of the reticule plate, counted from the meridian toward the left, be A Then

$$A = -(a+c)$$

There will be two values of c, one for clamp east and the other for clamp west. The one to be used is that corresponding to the position of the instrument when its middle wire was set upon the center of the reticule plate. If A exceeds one second of time for several nights, the plate-holder must be adjusted by moving it one-thirtieth of an inch for each second of error.

The method of determining the error of the third adjustment has been already described in section V.

XIII.—FITTINGS OF THE PHOTOGRAPHIC HOUSE

The photographers will examine the photographic house as soon as it is erected, and see that all white light is perfectly excluded. A single crack in the wall, or even an unprotected keyhole may cause irreparable mischief. Such openings are readily detected from the inside, and when found may be stopped up or covered with yellow paper or other suitable material. In full daylight the orange-glass window will admit too much light, and must be covered with orange envelope paper.

The emulsion chest, drying box, and plate boxes are also to be examined, and any cracks or other openings closed up. The drying box may be screwed to battens fixed against the side of the house, or, if more convenient, it may be mounted on legs. To facilitate cleaning, the top of the box lifts off, and the bottom may be removed by taking out a couple of screws.

Close to the left-hand side of the pier carrying the plate-holder a stand large enough to support a chronometer and the blank form for recording the exposure of plates will be required. A shelf long enough to hold two plate-boxes, side by side, will be fixed to the wall of the dark room close to the right-hand side of the pier, and at a convenient height for taking plates from, and returning them to, the boxes.

A screen should be interposed between the plate-holder and the boxes on the shelf, so that if the slide were moved while the boxes were open (which ought never to happen) no light could fall on the plates. That part of the opposite wall which receives the sunlight passing through the plate-holder should be blackened, or covered with dark-colored cloth, so as not to reflect light about the room.

If external objects illuminated by direct sunlight are visible from any part of the plate-holder in any position of the slide, foggy plates and blurred images may be expected, and under circumstances requiring long exposures any light-colored surface, exposed to radiation from the sky may produce similar effects.

An abundant supply of good water is indispensable. For developing dry plates almost any well, spring, or river water may be used, even if it contains rather large quantities of some salts and is not entirely free from organic matter. If it should be necessary to prepare silver baths, water must be distilled if it cannot be otherwise obtained of sufficient purity.

XIV.—CARE OF THE SENSITIVE EMULSION

The sensitive collodio-bromide emulsion is contained in bottles of orange glass, only partially filled to facilitate efficient shaking. After standing undisturbed for a long time the silver bromide partly subsides to the bottom of the bottle, but it may be perfectly re-emulsified by agitation. By inverting the bottle and looking through its

bottom toward the dark-room window, a more or less abundant deposit will be seen, and the shaking should be persevered in until this is entirely broken up and washed away When this has once been thoroughly effected the bromide will be easily kept in suspension by a little shaking at intervals of a day or two Collodion vials containing emulsion should always be well shaken and then allowed to stand a few minutes before coating plates As issued it contains a certain amount of coarse sediment which must be removed by filtration

The emulsion must not be exposed to white light, for although the bottles containing it are of colored glass they cannot be absolutely relied on to protect it from injury It is true that some collodion emulsions may be submitted for a short time to the action of weak daylight without material deterioration, but in other cases only foggy images can be obtained after such treatment Additional caution is requisite with emulsion that has been transferred to collodion vials of colorless glass The greatest care must also be taken not to contaminate the emulsion by flowing it upon glass that is not perfectly clean, or by putting it into bottles that have been exposed to light with traces of emulsion adhering to them

XV—SELECTING AND MARKING GLASS.

The photographic operations will be begun by examining the stock of glass and rejecting all plates that are broken or cracked, not sufficiently flat, with a rough or uneven surface, too large to enter the grooves of the plate-boxes easily, or having any corner so short that it will not rest securely on the pins in the plate-holder of the photoheliograph A sufficient quantity of the best glass, not less than 204 plates, is next to be selected for the transit plates, and after choosing the best surface for the front, or film side, each plate is to be marked in one corner of the back with a number, beginning with unity and proceeding consecutively upward This is to be done neatly and legibly with a writing diamond, and to facilitate reference to the finished photographs when stored in plate-boxes the numerals should be so placed as to be upright in the right-hand upper corner of the plate

XVI—CLEANING AND ALBUMENIZING GLASS

Remove the sharp edges with a whetstone, and clean the glass from any gross impurity that may be adhering to it Old films are best got rid of by soaking for a few hours in a moderately strong solution of concentrated lye, after which they can generally be washed away without much labor, but the surface of the glass may be injured by leaving it too long in such a solution, and especially by allowing the latter to dry upon the plate

The chromic solution recommended by Mr M C Lea is best adapted to the circumstances of the transit of Venus photographers, as it rapidly oxidizes organic impurities and gives off no vapor injurious to instruments

Solution for Cleaning Glass.

Bichromate of potash -	4 ounces
Sulphuric acid - - -	6 fluid ounces
Water - - - - - - -	50 fluid ounces.

Put the bichromate of potash into a two-quart bottle and pour in the water. Then add (out of doors) the sulphuric acid, a little at a time, shaking the bottle well and allowing it to stand a few minutes after each addition. It will become very hot and give off a little corrosive vapor, which, however, soon disappears. If too much acid is added at once the bottle will probably be broken by the heat. When the bichromate of potash is all dissolved, and the solution has cooled, it is ready for use. Pour it into a large rubber pan, and immerse, one at a time, as many plates as the solution will cover. If any bubbles are allowed to remain between the plates they will prevent the solution from acting on the surfaces in contact with them. The glass should remain in the chromic solution through one night at least, and a still longer time may be desirable, but it ought not to exceed a week, or thereabout.

Remove the plates from the cleaning solution and put them into a pan or bucket of clean water. The solution may be repeatedly used; it will not injure the skin beyond causing a slight stain and a rather disagreeable odor, but it is very destructive to clothing. Renew the water in the bucket, and separate the plates from each other until the yellow fluid adhering to them is entirely washed away; then refill the bucket so as to leave them entirely covered with clean water.

The plates are now to be taken one at a time from the bucket and rubbed with the fingers, or with a clean cloth, on both sides and around all the edges while the water from the tap flows on them. After a final rinse they will be ready for albumenizing.

Albumen Solution

White of egg - - -	1 fluid ounce.
Water - - - - - -	16 fluid ounces.
Strong ammonia - -	15 minims.

Put the white of egg into a clean bottle of convenient size together with a few pieces of broken glass, shake it vigorously until the albumen is thoroughly "beaten," and allow it to stand undisturbed for several hours. Then add the water and ammonia, and shake just enough to mix the contents of the bottle, after which it should again stand for some time before being filtered for use. If tolerably fresh eggs cannot be obtained, 100 grains of dried albumen may be taken as the equivalent of 1 fluid ounce of white of egg, but the latter is much to be preferred.

Filter the albumen solution into an 8-ounce graduate until it is two-thirds filled, keeping the neck of the funnel under the surface of the filtered albumen to avoid bubbles. The few bubbles that unavoidably form may be removed with a wisp of clean paper.

When a plate has been washed and rinsed as already described, observe by the mark on the back which is the front side, and flow it with the albumen solution. After draining the plate for a few seconds, flow it a second time precisely as before and set it on a rack to dry, always keeping the same corner downward, and never touching the front surface or the uppermost edges. The plates will be apparently dry in the course of an hour or two after albumenizing, but the albumen should be allowed to become thoroughly desiccated and hardened by keeping in a dry place, properly

protected against dust, for several days if possible. If coated with emulsion too soon after albumenizing, the films will be more liable to blister and rise from the glass during development, a misfortune to be avoided by the exercise of every possible precaution. The albumenized plates are finally to be examined, and any imperfect ones set aside to be cleaned again.

As the permanent marks on the plates will scarcely be visible in the dark room, write the number of each one with a blue pencil, or with ink, in large and plain figures on a gummed label, and attach it to the back of the corresponding plate in such position that when the latter is put into a plate-box the permanent mark will be seen in the upper right-hand corner, and the number on the label in the upper left-hand corner of the plate. Re-examine them carefully to insure that the two numbers on each plate are identical, and, if found so, store them in plate-boxes for use as required.

XVII.—COATING PLATES.

As the plates are necessarily exposed for a considerable time to whatever light may be in the dark room while they are being coated, it is necessary to proceed with caution. But by operating near the window while the emulsion vials, and the rack holding the plates already coated, are protected from direct light by interposing screens, it is possible to work with both ease and safety. The outer door must, of course, be locked. In very warm weather it is more comfortable to coat plates at night with the door and window open if there are no gas lamps or other dangerous lights outside. It will be safe to use naked candles if they are so screened that no direct light from them or from any nearly white object strongly illuminated by them can fall on the plates or the emulsion. Light from gas or coal-oil lamps is more actinic than that of candles, and must be used with caution if at all. Orange-glass lanterns ought to be quite safe if candles are used in them, but it must not be taken for granted that they are so.

Set one of the racks from the drying box where no direct light from the window can fall on it, and have at hand a wide camel's-hair brush, and a bottle containing a mixture of two volumes of ether and one volume of alcohol for thinning the emulsion. Begin by thoroughly shaking the bottle of emulsion it is proposed to use; then insert a tuft of filtering cotton in the throat of a perfectly clean emulsion filter and moisten it with a little of the thinning mixture. Set the filter in a twelve-ounce collodion vial, pour in emulsion from the bottle until it is nearly full, put on the cover, and set an orange bell-glass over the filter and vial to diminish evaporation and prevent the access of light. If the tuft of cotton is too large and too closely packed, filtration will be slow and tedious; if not properly inserted it will permit the passage of small particles of sediment, &c. The emulsion should pass through in a rapid succession of large drops, about one per second. It may run in a thin stream at first, as its passage will be slower when the cotton is fully saturated with emulsion. Replenish the filter from the emulsion bottle before it becomes empty, and transfer it to another collodion vial when the first one is two-thirds full. While plates are being coated from one vial the other will be filling, and so on, using them alternately.

Take a plate from the plate-box, and, holding it near the window, see if the

albumenized surface is free from fibers and particles of dust. Such substances may be carefully removed with the brush, but actual brushing will leave marks on the albumen. Holding the plate by the corner that was marked with the writing-diamond, coat it with emulsion, precisely as if it were collodion, and, as soon as the film is set, put the plate in the rack. As emulsion is, under ordinary circumstances, rather less fluid than collodion it is possible that operators who are accustomed to use thin collodion, and coat their plates very deliberately, may experience some difficulty in obtaining a uniform film. Pour on a rather liberal dose of emulsion and cover the plate quickly; then incline the plate but little, so as to pour off very slowly, and impart to it whatever motion may be necessary to prevent the formation of lines. The precautions usual with careful operators to prevent particles of dried emulsion on the lip of the vial from falling upon the plate must, of course, be observed. It is sometimes recommended to pour from one vial and to drain the plate into another, so that emulsion once poured out may be filtered before it is used again; but this mode of operating is inconvenient, and will not be necessary unless the atmosphere of the dark room is charged with dust. The corner by which the plate was held while coating with emulsion is, of course, left uncovered; by looking toward the window through this clear space the permanently marked number of the plate may be read in the dark room, though with some difficulty. When the emulsion becomes too thick to flow well (or sooner if particles of foreign matter are seen in it) add as much of the mixture of ether and alcohol as may be required to bring it to the proper consistency, pour the whole into the emulsion bottle, and shake it until well mixed. Then transfer the filter to the empty vial, and begin using the full one. The filter must not be allowed to become empty. If the vial containing it is getting too nearly full it may be set in the emulsion bottle. Do not dilute the emulsion unnecessarily by using the thinning mixture too freely.

When the rack is full of plates remove it to the highest unoccupied position in the drying-box, and avoid all further risk of accident to them by closing and bolting the doors. The box will contain 120 plates, but it is better to make a smaller number at one operation, filling only every second or third groove of each rack. When a sufficient number of plates are prepared return the bottle and vials to the emulsion chest, and wash the filter perfectly clean before the adhering emulsion becomes dry and hard.

Plates freely exposed to the air of the dark room would be dry in an hour or thereabout; in the drying-box a longer time will be required, and several hours at least should be allowed. At night the top and bottom of the box may be taken off. When quite dry the plates are to be put in plate-boxes that are clean and free from dust, in the order of their numbers, with the films toward the back or hinge side of the boxes, the numbered corners uppermost, and the lowest numbers in front. Wrap each box in thick paper, secure it with stout twine, and mark plainly on the package the numbers of the plates it contains.

With each batch of plates prepared for observing the transit at least two test-plates are to be made, one near the beginning and one near the end of the operation, using for that purpose some of the glass not selected for the transit work. The test-

plates are to be dried with the others and afterward exposed and developed If they prove to be good the rest of the lot is likely to be equally so

XVIII.—EXPOSURE OF PLATES IN THE PHOTOHELIOGRAPH

The sun's image, as seen on the reticule-plate of the photoheliograph, generally has an irregular vibrating motion in different directions, arising from various causes As the exposure of different parts of the plate lying in the direction of motion of the exposing slide is not simultaneous but successive, any movement of the image as a whole will produce a distortion in the resulting photograph symmetrical with respect to its vertical diameter, and inversely proportional to the velocity of the slide Moreover, the limb itself is in a state of constant and rapid local agitation, which in some conditions of the atmosphere is so exaggerated as to produce the boiling or flaming appearance familiar to all observers It results from this that, quite apart from any distinctively photographic effect, the longer the exposure the larger the photograph will be, and that if the motion of the slide is not uniform the limb on the side where the velocity was least will be extended more than the other So brief is the time of exposure that these effects are indeed minute, but it cannot be assumed that they are in all cases inappreciable It follows, therefore, that a given exposure is more advantageously made with a wide opening of the slit and a rapid motion of the slide than with a narrow opening and a slow motion, and that the velocity should be in all cases as nearly uniform as possible

It is expected that the chief astronomer will himself expose the plates on the day of the transit, or, at least, that he will supervise the manipulation of any person to whom this duty is delegated, and see that such assistant has the requisite instruction and previous practice The movement of the slide should be as rapid as can be given with ease and uniformity, retaining control of it throughout, and avoiding any approach to violence Toward the end of its course the motion should be slackened so as not to endanger the stops, but it should continue until the slide rests against them As the regulation of exposure depends entirely on the uniformity of this movement, it should be practiced until a fixed habit is acquired It is of course important that there should be no great difference in velocity, whether the motion is from east to west or from west to east To increase the exposure, widen the slit by separating the sliding plates, to diminish it, bring them nearer to each other, always setting them so that the center of the slit shall be in line with the middle pair of screws The milled nuts must be screwed up so as to clamp the plates securely without using too much force

In exposure considerable latitude is allowable, indeed the appearance of the sun's image is so similar with very different exposures that the real importance of this element is liable to be underrated The planet requires less exposure than the sun's limb, but good definition of the latter is the principal end to be attained If with chemicals in good order and proper development the image comes out reluctantly and remains very thin, or if there is any material falling off in density near the sun's limb, the exposure is too short To find the correct exposure, begin with one known to be sufficient, and gradually lessen it until signs of under-exposure just begin to appear If the image is distorted, or looks as if the plate-frame were out of focus, the defect is

most probably due to flexure of the mirror. The slightest tension arising from improper mounting, either of lens or mirror, will make itself apparent in this way.

XIX.—THE DEVELOPMENT.

The water used for washing the plates before development, and the developer itself, should have a temperature of at least 60° Fahr.; 90° to 100° is still better. If the weather at any of the northern stations should be cold, it will be well to have a small supply of warm water at hand for these purposes.

The following solutions will be required:

Alcohol and tannin.

Tannin - - - - - - -	20 grains.
Strong alcohol - - -	1 fluid ounce.
Water - - - - - - -	1 fluid ounce.

Pyrogallic solution.

Pyrogallic acid - - -	3 grains.
Water - - - - - - -	1 fluid ounce.

This solution is decomposed by keeping, and only so much must be made at one time as can be used immediately.

Dilute ammonia.

Strong ammonia - -	30 minims.
Water - - - - - - -	1 fluid ounce.

Bromide solution.

Bromide of potassium -	20 grains.
Water - - - - - - -	1 fluid ounce.

To these may be added:

Alkaline citrate solution.

Citrate of ammonia - -	30 grains.
Strong ammonia - - -	30 minims.
Water - - - - - - -	1 fluid ounce.

The dilute ammonia, bromide, and alkaline citrate are to be transferred for use to dropping-bottles, so conspicuously labeled that they can be easily distinguished from each other in the dark-room. It must be borne in mind that a drop from one of the tubes is much smaller than one from the lip of a bottle.

Having put one fluid ounce of pyrogallic solution into one of the small, wide-mouthed bottles issued as developing glasses, begin by treating the film with alcohol and tannin. One principal object of this application is to harden the substratum of albumen. The solution should therefore be flowed back and forth over the plate for at least a minute,

and be returned to the bottle when the film is thoroughly saturated. The manipulator is next to be wetted and applied to the back of the plate unless the operator, fearless of stained fingers, prefers to dispense with it. Wash the plate under the tap, or with warm water if necessary, until the water flows smoothly over the film. Then add two drops of bromide solution and two drops of dilute ammonia to the pyrogallic solution in the developing glass and apply it to the plate, keeping it in gentle motion over the film to promote equal development. The image should appear quickly and gradually increase in strength. Subsequent additions of dilute ammonia are to be made, a couple of drops at a time, as the appearance of the plate may indicate, accompanying every alternate addition with an equal quantity of bromide.

If it is preferred to use the alkaline citrate, the developer is to be prepared at first as given above, and the subsequent additions will be of the alkaline citrate solution only. This treatment has given excellent results in some cases and is provisionally recommended.

The formula given above is offered as a starting point for such modifications as circumstances may require, and not as an absolute standard of universal application. The character of the image sought, the age and quality of the emulsion, etc., are varying conditions which render impracticable the adoption of any rigid rules. The end in view is, by means of a developer strong in pyrogallic acid and weak in ammonia to bring out an image of equal density throughout, while the rest of the plate is kept clean by a sufficient quantity of bromide, or of bromide and citrate of ammonia. A greater strength of pyrogallic acid than that prescribed can hardly be required, but this solution may sometimes be advantageously diluted. If the image come out slowly it must be allowed to take its time, and not be forced with too much ammonia. It is scarcely possible to produce dense fog, but a veiled image is usually the result of an excess of ammonia. Operators accustomed to the development of gelatine plates must be especially cautious in this respect.

The finished photographs must belong to one of two classes. It is not possible in the case of the sun to obtain an outline as sharp and distinct as in photographs of the moon or of terrestrial objects. But the nearest possible approach to such a definite line is precisely what will most facilitate measurement of the plates, and this must be sought as being of much greater importance than the attainment of any special standard of density. An image that is dull and blurred, when held over a black surface and seen as a positive by reflected light, and that is thin and hazy when viewed as a negative by transmitted light, will be almost worthless, while one bright and clear as seen in either one of the two ways will be valuable. If, therefore, the operator finds it easy to produce a sharp and clear image of the ambrotype variety he may do so, taking care, of course, not to discontinue the development until quite certain that it is equal all around. But if there is from any cause a tendency to discoloration of the film, giving a dull appearance by reflected light, a greater degree of density will be requisite. A slight veiling, even, is not very injurious if the image is clear and strong by transmitted light. No effort should be wasted in striving to attain an unnecessary degree of density, for, although it can hardly be too great, a very moderate density will suffice.

XX.—FIXING AND VARNISHING

As soon as development is complete wash off all traces of the developer and with a weak solution of hyposulphite of soda. Cyanide of potassium must not used.

Fixing solution.

Hyposulphite of soda, - - - -	½ oz. to 1 oz.
Water, - - - - - - - - - - -	16 fluid ounces.

As the silver bromide dissolves rapidly, it is most convenient and safest to pour little of the fixing solution on the plate, flow it about until the film is clear, and final dismiss it into the sink. Then wash the plate thoroughly and set it in a rack to d After the lapse of sufficient time, several hours at least, varnish in the usual mann

XXI.—BLISTERING AND LIFTING OF THE FILM

If the film separates from the glass, or if small blisters form near the sun's lim the planet, or the plumb-line, the plate is worthless, and no precaution must be ne lected that can tend to prevent so great a misfortune. A substratum of albumen th is too thick, such as would be obtained by using a solution materially stronger th that of the formula given above, may be imperfectly coagulated by the alcohol a tannin, and thus aggravate the evil it is designed to prevent. If the directions alrea given for drying the albumenized plates thoroughly, and for saturating the film w the solution of alcohol and tannin, are followed, it is probable that no trouble will experienced, but should a tendency to this defect manifest itself during the prelimina practice, special caution will be necessary in conducting the development of the tran plates. Too much ammonia will act on the substratum and must be avoided. T development must be stopped as soon as the least allowable density is reached, a the plate must be fixed by pouring on the hyposulphite solution, and not in a pan.

XXII.—SPOTS AND OTHER DEFECTS

Small circular transparent spots may appear on the sun's disk, which, if numero and of a certain size, might resemble the image of Venus and cause time to be lost measuring the plates. The emulsion furnished is remarkably free from any tenden to produce these spots. If they appear it will be from one of two causes: 1. Partic of foreign matter in the emulsion; 2. Particles of dust that have settled on the pl after coating, especially if charged with chemical substances, such as might arise fr solution of bichromate of potash spilled on the floor of the dark-room. The form cause may be removed by properly filtering the emulsion; the latter must be avoid by cleanliness and greater general care.

Certain other defects, such as pin-holes, crapy lines, &c., do not materially detra from the value of the plate, but every operator who cares for his reputation will nev theless seek to avoid them.

XXIII.—PREPARATORY PRACTICE

To familiarize themselves with all the details of their work and with the materi they are to use, the photographers will begin to prepare, expose, and develop plat

in accordance with the foregoing instructions, as soon as possible after arriving at their station, and will continue such exercise until the chief of the party is satisfied that they can produce solar photographs of satisfactory quality with ease and certainty They will avoid any unnecessary expenditure of materials, and will take care that a sufficient supply of everything is reserved for the operations of the transit day If the bottles of emulsion are labeled with different letters it signifies that pyroxylines of slightly different character have been used Test the contents of each bottle, and if notable differences of quality are found to exist among them, reserve the best for the transit plates, or mix the various kinds judiciously, as may appear to be preferable

It has been already remarked that the person who is to expose plates during the transit must acquire a fixed habit of manipulating the exposing slide, and that unless an approximately uniform motion can be thus obtained there will be no means of regulating the exposure Having secured a tolerably uniform action of the slide by whatever practice may be requisite, proceed by trial according to the instructions given for ascertaining the correct exposure, to find the proper opening of the slit when the sky is quite clear and the atmosphere in the most favorable state This will be the least opening that can be used at any time, and the one with which the observation of the transit will be commenced if the weather should be good

If the sun is obscured by clouds so as to be visible for a few minutes only during the transit, it will be necessary to work as rapidly as possible The manual of operations prescribed for the observation of the transit will permit plates to be exposed at intervals of only a few seconds if every one is perfectly familiar with his duties In order that each may be prepared to perform his part of the work promptly, the whole party must be drilled from time to time, going through with all the details of making the record, &c, except that plain glass will be used instead of sensitive plates, and there will, of course, be no development

XXIV—PREPARATION FOR THE TRANSIT

When the photographers have become sufficiently familiar with the process they are to use, they will begin to prepare a stock of dry-plates to be used in observing the transit Eighteen plate-boxes, capable of holding twelve plates each, are supplied to each party Seventeen of these are to be filled with dry-plates, leaving one box empty The plates are to be arranged in the boxes in the order of their numbers, as already directed, and the boxes must be so marked that the plates can be exposed in consecutive order The remaining glass should be cleaned and albumenized, to be used with wet emulsion in case of emergency, and a sufficient quantity of the developing solutions for fifty or more plates must be provided

Everything belonging to the photoheliograph must be in correct adjustment and in good working order, especially the clock-work of the heliostat, which must be so well regulated that it can be left to itself for several minutes without allowing the sun's image to get too far from the center of the plate See that the mirror, lens, and reticule-plate are perfectly clean, and remove any dust that may adhere to the last-named by wiping it with chamois leather If artificial light is required for reading the chronometer and making the record, as it probably will be, an orange-glass lantern must be used.

XXV.—PHOTOGRAPHING THE TRANSIT.

Photographs are to be taken only while Venus is completely within the limb of the sun. When the chief astronomer decides that the planet has progressed sufficiently far upon the sun's disk, the automatic break-circuit key of the exposing slide will be put in circuit with the chronograph, that instrument will be started, the members of the party will repair to their respective stations, and, after locking the outer door of the photographic house, the exposure of plates will begin. The services of four persons will be required; if the party consists of but four, their duties will be as provided in what follows:

The assistant astronomer will be stationed at the heliostat, where he will watch the image of the sun on the target of the exposing slide, and keep them nearly concentric by an occasional movement of the tangent screws. The image must never be allowed to get so far from the center as to partly uncover the black disc of the target. He will also note the condition of the sky, and give prompt warning to those in the photographic house when the sun is obscured by clouds, and when it reappears. He will occasionally see that the chronograph is working properly, and give notice when it must be stopped to renew the paper. And finally, he will read the barometer and thermometer as directed in another part of these instructions.

The chief astronomer will make the exposures and keep the record in the following form:

RECORD OF PLATES EXPOSED DURING THE TRANSIT OF VENUS, DECEMBER 6, 1882

At.... ....; *Chronometer*..................

Plate exposed.	Time of exposure by—	Time of exposure by—	Plumb-line pointer.	Slide moved.	Temp. Fahr.	Remarks.
	Chronometer.	Chronograph.				
No.	*h. m. s.*	*h. m. s.*			°	

When the number of a plate is called he will enter it in the first column, and, taking the time from the chronometer, he will make the exposure at some beat of the latter, and, after giving the chronograph signal, or "rattle," with the break-circuit key, he will enter the chronometer time of exposure in the second column. The time recorded by the chronograph will be read off, and filled in, subsequently.

In the column headed "plumb-line pointer" he will write E. or W., as the case may be. The pointer should be frequently reversed, but if the change is made after the exposure of every plate, the plumb-line will probably never be quite at rest. A much better way is to expose the plates in groups of six, the plates of a group following each other in rapid succession. Then reverse the pointer and give the plumb-line time to come to rest before exposing the next group of plates.

In the column headed "Slide moved" write E. or W., according to the direction of that motion. The movement should be alternately eastward and westward, and the slide must always be left resting against the stops that limit its course.

Record the temperature of the photographic house in its proper column at intervals of half an hour, or more frequently if the change is rapid

In the column for remarks any peculiarity of the plates, of the exposure given, or of atmospheric conditions, and any other circumstance likely to affect the result, should be noted

When the chief astronomer desires a pause in the work to follow the exposure of any plate he will notify the photographer who changes the plates of his intention before giving the exposure, so as to avoid unnecessary handling of the plates

When a plate has been developed he will examine it, and, with the advice of the photographers, decide whether any change in the opening of the slit is desirable

One of the photographers will put the plates into the plate-frame, and remove them after their exposure He will commence operations by placing the empty plate-box on the right-hand end of the shelf erected for that purpose, and the box containing the plates that bear the lowest numbers of the series, beginning with No 1, at the left-hand end of the shelf When directed to proceed he will open the left-hand box, take out the nearest plate, which should, of course, be No 1, carry it to the plate-frame, keeping the same edge of the plate uppermost, and carefully avoiding any contact with the plumb-line After securing the plate with the curved spring he will close the plate-box, and then call the number of the plate, reading it from the label, which will be in the upper left-hand corner of the plate-frame He will then watch the back of the plate closely to see that the sun's image is not too near the edge of the plate If the distance should be less than about an inch he will so report to the chief astronomer

When the image appears for an instant on the plate he will open the right-hand box, take the plate from the plate-frame, carry it to the farthest groove in the box, and then close the box Without waiting for orders he will then open the left-hand box, take out the nearest plate as before, and so on, always keeping the back of the plate toward himself, and the numbers uppermost, and always replacing the plate in the farthest unoccupied groove of the right-hand box

When the right-hand box is full the other will be empty, the full box must then have a gummed label pasted over the hook in front, so that it cannot be easily opened, and must be put in some place specially designated for the reception of exposed plates Then remove the empty box to the right-hand end of the shelf, place the next box of the series to the left of it as before, and so proceed until all the plates are exposed To avoid mistakes, the boxes should be arranged beforehand so that they will be taken up in consecutive order

The other photographer will develop plates during the whole time of the transit, beginning with the one first exposed As each plate is fixed and washed he will examine it carefully to see if the exposure is correct, and will call the attention of the chief astronomer to any alteration that may be required He will then develop another plate, always taking the one last exposed The grooves in the boxes belonging to plates taken out for development are to be left vacant, and not filled by other plates

If the sky is clear the groups of plates exposed should be equally distributed over the whole time of the transit, but if it is probable that the entire transit will not be seen, a considerable number of photographs should be secured at once, reserving

some plates to be used if the weather should prove better than was anticipated If the sun is visible at intervals between passing clouds, every one must be at his station to take instant advantage of every opportunity that may offer

The sun may be partially obscured by thin clouds or by a hazy atmosphere, so as to require very long exposures After the slit has been opened as widely as possible the exposure can be still further increased by a slower movement of the slide, but good results can hardly be expected under such circumstances

XXVI—DEVELOPMENT AND PACKING OF TRANSIT PLATES

As soon after the transit as possible the photographers will begin to develop the transit plates, and will continue that work without unnecessary intermission until it is completed After drying and varnishing the plates they will be put in the plate-boxes, and kept from shaking about by small rolls of Joseph paper, long enough to reach crosswise over the tops of the plates, and just large enough to hold all the plates firmly, without too much force, when pressed down by the lid of the box Two such rolls will be put in each box The boxes are then to be covered with strong paper and tied with stout twine

In separating the plates into two or more lots, to be forwarded at different times to Washington, the plates of each lot should be selected so as to include all periods of the transit If there are only two lots, one should contain all the even-numbered plates, and the other all the odd-numbered ones

XXVII—WET EMULSION PLATES.

Wet plates may be prepared with emulsion if any accident should cause the loss of a great part or all of the dry plates when it is too late to replace them A glass plate cleaned, albumenized, and coated with emulsion, as already directed, is to be immersed in clean water contained in a pan or dipping-bath as soon as the film is set When the water flows smoothly over the film, as the plate is lifted, the exposure may be made, but the plate may remain in the water for any reasonable length of time without detriment Such plates are developed precisely like dry ones, omitting the preliminary treatment with alcohol and tannin They develop more rapidly than dry-plates made from the same emulsion

XXVIII—BATH WET PLATES

Collodion shrinks greatly in drying, but when once dry it swells but little if wetted either with alcohol or water A film that has been dried before it is exposed is to be preferred, therefore, to one that is exposed while wet, for photographs that are to be accurately measured And dry-plates can be exposed so rapidly in the photoheliograph that if the sun should be visible for a short time only during the transit, a most important advantage would be gained by employing them For these and other reasons, it is expected that they will be used But if from accidental loss of materials, from lack of experience, or from any other cause, the photographers are unable to prepare and develop emulsion dry-plates successfully, they will have recourse to the usual bath process It must be definitely decided in advance which process will be used, and preparations must be made for that one only.

If a resort to the bath wet process is deemed advisable, the services of at least one additional photographer must be secured if possible. With three or four photographers it will probably be best to have the chief photographer collodionize all the plates and immerse them in the baths, keeping his hands clean. The other photographers will withdraw the plates from the baths and drain them, put them in the plate-frame, and call their numbers. After exposure they will remove the plates from the plate-frame, develop, fix, and wash them, and, finally, put them in the drying racks. Each photographer will go through the whole course of those manipulations with every plate he takes from the bath, and the different operators will follow each other as rapidly as the limited accommodations of the photographic house will permit.

The best collodion is one that gives a rather hard and patchy negative, for softness, and correct rendering of light and shade, are rather to be avoided than otherwise. The following formula is recommended:

Bromo iodized collodion.

Alcohol - - - - - - -	10 fluid ounces.
Ether - - - - - - -	10 fluid ounces.
Iodide ammonium - -	40 grains.
Iodide cadmium - - -	60 grains.
Bromide ammonium - -	20 grains.

At least three silver baths will be required. They should contain 40 grains of nitrate of silver to the ounce of water, and enough nitric acid to redden blue litmus paper slowly. On removing the plates from the bath drain them as thoroughly as is consistent with rapid work, and wipe their backs. Develop with

Developer.

Protosulphate of iron - -	1 ounce.
Acetic acid No. 8 - - -	$1\frac{1}{2}$ fluid ounces.
Water - - - - - - - -	20 fluid ounces.

These proportions may be varied, however, if it is found advisable to change them. Fix with solution of cyanide of potassium of such strength as to clear the film rather quickly.

The instructions already given for the manipulation of dry-plates will be followed so far as they are applicable. If bath wet plates have been exposed in the photoheliograph, the plate-frame must be thoroughly cleansed from all traces of nitrate of silver before dry-plates are put into it.

XXIX.—GENERAL PRECAUTIONS.

Photographers who are accustomed to the use of dry-plates will be careful from habits already formed, but those who have practiced only the usual wet process must be very cautious indeed to avoid the loss of plates by accidental exposure to light.

Plate-boxes must be opened only to put in or to take out plates, and must be closed again as soon as possible. The drying-box must never be allowed to stand with open doors to avoid the inconvenience of opening and closing them frequently.

Emulsion bottles and vials must be replaced in the emulsion-chest immediately after using them, even if they will soon be wanted again. The lid of the chest should be secured by a strap or otherwise, so that it cannot be left open. The chest must be kept locked, with the key in some specially designated place and not in the key-hole.

When filtering emulsion, and while making or exposing plates, the outside door of the house must invariably be locked.

And, finally, the various manipulations that have been described in these instructions must be conducted with constant and scrupulous regard to cleanliness. It is only by conscientious attention to details like the foregoing, which are by no means trivial, that successful results can be confidently anticipated.

XXX.—THE CHRONOGRAPH.

The speed of the chronograph is governed by a vibrating spring whose normal rate of motion is one hundred and thirty-two vibrations per second. In regulating it, the time of revolution of the cylinder must be made correct within about two per centum by moving the sliding weight near the root of the spring, and then the final adjustment can be effected while the instrument is running by slightly loosening or tightening the capstan-headed screw confining the cheeks between which the spring is held.

The speed of the train is so great that to avoid detrimental friction all the pivots must be frequently oiled, but especially the escape-wheel pivot, and those near it. Weights of one hundred, fifty, twenty-five, and twenty-five pounds are furnished with the instrument, but in ordinary summer weather one hundred pounds drives it well. In winter, a little more may be needed. The weight should have space to fall three feet. This suffices to run the instrument two hours, and if it is wound when a fresh sheet of paper is put on the cylinder, it will not require winding again till the paper is changed.

XXXI.—TIME OBSERVATIONS AND CHRONOMETER COMPARISONS.

Should the station not be in telegraphic communication with a fixed observatory from which local time is received, two azimuth stars above the pole, two below the pole, and six or eight time stars should be observed with the transit instrument on every night when it is practicable. One-half the observations of each class should be made with clamp east, the other half with clamp west. Should the observer be able to get his local time from a fixed observatory, his own determinations may be omitted when not necessary to the success of the expedition. They must, however, be carefully attended to, so far as may be required, either for the determination of his own longitude, or for comparing his local time with that of other parties in the neighborhood. In any case, enough observations must be made to determine the azimuth of the photoheliograph and detect any changes that may occur in it.

All chronometers at the station must be compared daily, when they are wound, unless the local time and longitude are so well determined that no interest attaches to their running. It will sometimes be necessary to carry one of them about,

but the others should never be moved when it can be avoided. Every care should be taken to keep them at as uniform a temperature as possible, and the degree of heat to which they are exposed should be noted and recorded three or four times a day.

XXXII.—EXCHANGE OF TIME WITH OTHER PARTIES.

Should any opportunity offer for the comparison of local time or chronometers with parties from other countries, it must be improved. In such case, the observer must be careful to keep a complete copy of the comparison, and to assure himself that he has all the data necessary for determining the difference of longitude between the stations compared.

XXXIII.—LATITUDE AND LONGITUDE OF STATION.

The latitude of the station must be determined with the meridian instrument, used as a zenith telescope, and not less than thirty-six observations should be made upon at least twelve pairs of carefully-selected stars.

Special attention must be paid to getting the true longitude of the station, but the best method of doing this will depend upon circumstances. If the station is in a region covered by an accurate trigonometrical survey, or if it is in telegraphic communication with a fixed observatory, the determination of its longitude will be comparatively easy. In any case, the observer must be on his guard against depending upon a single result. If accurately known trigonometrical points are available, the position of the station must not be determined from a single one of them, but from at least three, whenever possible. If telegraphic signals are exchanged with a fixed observatory, the exchange should be continued through not less than three evenings. At places where neither the trigonometrical nor the telegraphic method is available, recourse must be had to occultations and moon culminations. In observing the latter, care should be taken that the number of observations before and after full moon are nearly equal, and that in each class about as many observations are made by the assistant astronomer as by the chief of party. Instructions respecting occultations are given in section XXXV.

XXXIV.—THE EQUATORIAL TELESCOPE.

At stations where it is necessary to observe occultations, the mounting of the equatorial must be commenced as soon as possible, taking precedence of that of the photographic piers. The site selected must be such that the instrument commands a good view of the eastern and western sky, to within five degrees of the horizon if practicable.

Caution.—The shade glasses supplied with the double-image micrometer are so constructed that they can be employed tolerably safely upon a bright sun with the full aperture of the telescope, but as a matter of precaution, they should be warmed a little before using them, and the telescope should never be left pointed at th

once, to the great danger of the observer's eye. For that reason they must never be used apart from the Herschel solar prism.

XXXV.—OCCULTATIONS.

At stations whose longitude is not otherwise determined, all visible occultations of stars by the moon which occur during the stay of the party, must be carefully observed. To facilitate this work, the instants of the emersions which happen after the full moon may be computed in advance.

From the time the new moon first becomes visible until her full, she is to be carefully watched with the telescope to see what stars will be occulted. These can be recognized from thirty to sixty minutes beforehand by remembering that the course of the moon is nearly at right angles with the line joining her cusps, and that she moves nearly her own diameter in an hour. Whenever there is a chance of seeing an occultation, a map of the relative positions of the moon and the stars in its neighborhood must be made; and if the occultation is actually observed, the exact point of the moon's limb at which the star disappeared must be noted on the sketch. Nothing must be recorded as an immersion or emersion except the actual sudden and distinct disappearance or re-appearance of the star at the moon's limb. If the star is lost in the moon's rays at that moment, the fact should be stated. Every observation must also specify the maker's name and number of the time-piece employed, and whether the occultation was well observed, and if not, what amount of uncertainty attaches to it.

Great care must be taken to guard against errors of 10^s in the record, and, to this end, it will be well to have an assistant call aloud every tenth second, "0," "10," "20," etc.

The chronograph may be used in observing occultations, but in that case a fraction of a second will be required for the observer to become conscious of the phenomenon, and to give the signal, and this interval must always be estimated by the observer, and recorded in the memorandum-book. But, the chronograph should never be trusted to exclusively, and, when used, either the observer himself or his assistant should note the chronometer time of the occultation, or of the signal with the key.

XXXVI.—GENERAL INSTRUCTIONS RESPECTING OBSERVATIONS OF CONTACTS.

The first question which the intending observer of contacts has to consider is whether the appliances at his disposal and the circumstances in which he is placed will permit of his making observations of astronomical value. If they do, especial pains and minute attention must be devoted to all the necessary preparations. The following is an outline of the general plan of operations:

Determination of time.—The most essential requirement is that the observer shall have the means of determining his local time within at least one or two seconds At fixed observatories there need be no difficulty in this respect. For the benefit of observers at other points it is intended to make arrangements with the Western Union Telegraph Company to transmit time-signals from the Naval Observatory to every part of the country. Individual observers who receive their time in this way should communicate with the authorities at the nearest telegraph station, and, in the event of

any doubt, address the Superintendent of the Naval Observatory, Washington, on the subject. Detailed information and instructions for receiving and understanding the time-signals will be sent to all who desire it in advance of the transit.

Size and quality of telescope.—The aperture of telescope to be preferred in the observations is from 5 to 6 inches. In order that all observations may be as nearly as possible comparable with those made in the Southern hemisphere, it is recommended that observers with telescopes exceeding 6 inches in aperture reduce the aperture to 6 inches in observing all the contacts. Apertures as small as 4 inches may be used without seriously detracting from the accuracy of the observation. Below 4 inches the value rapidly diminishes, and 3 inches may be regarded as the smallest with which observations of real value can be made. It is important that the objective should be of good quality, forming round, neat images of stars, with a power as high as 200. To test the objective, alternately push the eye-piece in and draw it out so that the star shall present the appearance of a disk of light. If the objective is good, this disk will be round and of uniform brilliancy; if the disk is irregular in outline, with permanent bright or dark regions in it, it shows the telescope not to be good.

Magnifying power.—The eye-piece should have a magnifying power not less than 150 nor much more than 200. Between these limits the choice may be regarded as indifferent.

Mounting.—An equatorial mounting is nearly indispensable to an accurate observation. Observers practiced in the use of an altazimuth mounting may possibly make an observation with one of that class, but they must be able to keep an object in the center of the field. A clock-motion is desirable, though not indispensable. If there is no clock-motion the telescope must be firmly mounted, and the observer must be well practiced in moving the eye-piece steadily with his hand so as to keep an object in the center of the field.

Micrometer.—A regular filar micrometer will not be of any use as an instrument of measurement, but spider-lines of some sort are desirable for the double purpose of insuring an accurate adjustment of focus and of estimating the brilliancy of the sun's disk. If the telescope is not supplied with a micrometer the observer should have a positive eye-piece, in the focus of which he should insert a spider-line or, better yet, if he is able to do it, a pair of spider-lines at such a distance that the angle between them shall be 1″ or 2″ of arc. In a 6-foot telescope the required distance will be about $\frac{1}{2000}$ of an inch. The observer should find by previous trials on the sun and stars the exact point when the spider-lines are in the focus of the objective so as to insure their being in proper position on the day of the transit. This point may be indicated by a mark on the eye-piece.

Shade glasses.—The common sun-shades, consisting of a single piece of thick glass, are very apt to split, and thus endanger the observer's eye, if the rays of a bright sun are concentrated upon them by an objective of larger aperture than two inches for a focal distance of thirty inches, or three inches for a focus of five or six feet. By making the shade of three thicknesses of glass, the piece next the eye being thickest and darkest in color, the other pieces being successively thinner and lighter in color, and all being fitted loosely into their cell so that they can

expand freely, it will be possible to use with safety an aperture of five inches upon a telescope of six feet focus. It is, however, recommended that, wherever possible, some other means of diminishing the sun's light be employed. Silvering the objective might be recommended, except for the possibility of cutting off too much light in a hazy atmosphere. The polariscopic eye-piece is commended for its convenience in use. If the observer cannot avail himself of it, a diagonal eye-piece with a reflector of plain unsilvered glass is recommended. In such an eye-piece the reflector is placed a little in front of the focus at an angle of 45° with the axis of the telescope. Being unsilvered, 92 per cent. of the light passes through it, and should be permitted to leave the telescope through an opening so as not to heat the air or the reflector. The remaining 8 per cent. of the light is reflected from the two surfaces of the glass. In order that these two systems of reflected rays may not cause confusion, the glass should be ground wedge-shape, and so arranged that only the reflection from the first surface may reach the eye. Since 4 per cent. of the sun's light will in nearly all cases be too great for the eye, the observer should also be provided with shade-glasses to still further diminish it. A neutral tint is to be preferred for all such glasses.

Day of the Transit.—It is essential that every observer intending to make a really accurate observation should have little else to attend to during at least an hour or two before the first contact he is to observe, and should be entirely free from visitors and inquirers. The points to be particularly attended to are the firmness of the telescope, his ability to move it in such a way as to keep any required part of the sun's limb steadily in the center of the field, and the accuracy of the focal adjustment. A mere estimate of an accurate focus about the time of observation should not be trusted to if it can be avoided, because the eye itself is liable to change its accommodation in this respect. The surest course is to have a pair of spider-lines previously fixed in the astronomical focus and to adjust the eye-piece so that these lines shall be sharply defined on the sun's disk. The observer can then be certain that his focus is right so long as the definition of the wires continues good.

The degree of brilliancy of the sun's disk as seen by the eye is to be particularly attended to. It was recommended by the Paris International Conference that the disk should be darkened to the point at which a pair of spider lines 1″ apart could just be seen distinctly separated. But as this test may not suffice, and as the observer may find insuperable difficulty in fixing the wires so close to each other, some other tests should be employed. We may lay down limits as follows:

I. If the brilliancy of the disk is such as to be at all unpleasant to the eye, or if there is any appearance of glare* surrounding the sun's limb, then the light is too bright and must be diminished.

II. If there is any difficulty in seeing the limb well and brightly defined, then the light is too faint. Perhaps a good rule will be to shade off the light to such a degree that with the center of the sun in the center of the field, the whole field is as bright as the observer finds it not unpleasant to look at continuously, and yet not so bright as to render the mottling of the photosphere indistinct. Then, since the sun's limb is

* This word is here used, not in the sense of a general atmospheric illumination, but in the sense of such a refulgence as to produce an appearance of indistinctness of outline through that excitation of the eye itself known as *dazzling*.

less than half as bright as the center of its disk, it may be presumed that the latter will be about of the right shade. But it must always be remembered that the slightest glare indicates too great a brilliancy.

Yet another guiding rule will be that the most distinct and easy view of the sun's limb is to be aimed at.

External contact.—To make a really good observation of this contact two conditions are essentially necessary in addition to all which have been described. The observer must have had some previous practice in observing first contacts, and he must know exactly where to look for the contact.

The first condition can be well fulfilled by the artificial transit of Venus apparatus, of which it is intended to have one or more in Washington available for observers.

For the second condition it is essential that the observer shall have the means of setting the spider-lines in the field of view to any required angle of position. Within the United States the first contact will occur at a point of the limb found by measuring 147° from the north point towards the east. The spider-lines should be set at right angles to that radius of the solar disk which terminates at this point of the limb. Then cutting off a segment of the disk by the spider-line, first contact will be seen in the middle of this segment. The tabular time of first contact at any point on the earth's surface may be found within a minute by subtracting 21 minutes from the time of internal contact. The interval between these contacts may be found with yet greater precision from the tables in the American Ephemeris for 1882. The tabular Greenwich time of internal contact may be taken at sight from the proper map accompanying this paper. Within the United States the tabular time of first contact may be regarded as 20 hours 55 minutes Washington time. In civil reckoning this is five minutes before nine a. m. Owing, however, to the errors of the tables, which observations of the transit will help us in correcting, it is quite possible that first contact will occur a large fraction of a minute earlier than the predicted time. To allow for this possible error, it is recommended that the observer begin to look exactly one minute before the computed time. The following is a specimen of part of the computation which the observer should make by the aid of the chart in order to know when to begin looking. Suppose the place to be Cincinnati. We find from the chart:

	h	m	s
Greenwich mean time of internal contact	2	24	52
Longitude of Washington	5	8	12
Washington mean time	21	16	40
Reduction to external contact		21	15
Washington time of tabular contact	20	55	25
Washington time to begin looking	20	54	25

The observer should avoid looking before this time in order not to fatigue his eye.

The time to be noted is that at which the notch made by the advancing planet first becomes visible. The observer may have to wait a few seconds to be sure that what he sees is really a permanent notch, but the time to be given is that when it was

first certainly seen. If he did not catch the first moment when he could see it, that fact must be stated.

Internal contact.—Owing to the importance of this observation and the necessity of special attention to it, it is recommended that the observer have little else to attend to during the 21 minutes between it and external contact. It is now believed that measures of the cusps with a double-image micrometer should not be undertaken during this interval, owing to their fatiguing the eye and distracting the attention of the observer.

It is essential that the observer should allow his eye nearly perfect repose for several minutes before the contact. It is quite right and proper that he should take a general view of the phenomenon at short intervals, and note the appearance presented by the outline of the planet, but he should not exercise his eye and attention in endeavoring to make any difficult observation.

His serious attention will be first required some two minutes before the expected time of contact. There is every reason to believe that the entire outline of the planet will then be visible, that portion not on the solar disk being bounded by a fine line of light, supposed to be due to the refraction of the atmosphere of the planet. Indeed, this line may be visible from the first moment of the planet's appearance, and the changes which it undergoes in the relative brilliancy at different points will be a subject of great scientific interest. Although observers of accurate contacts must guard against fatiguing their eyes by minute observations on this arc of light, observers who have not the appliances for making the best observations of contact might well devote themselves to its careful study.

One of the great difficulties in the observation of internal contact will be to avoid confusing this line of light, which may grow brighter as contact approaches, with the direct light from the sun's limb, which will be seen after contact. The distinction of the two is a matter of judgment which must be left with the observer. In what follows we, for the most part, make abstraction of this appearance, describing phenomena as if it were not present, and the observer must in like manner seek to observe as if it were not present.

The moment to be observed as that of true internal contact is when the limb of Venus is exactly tangent to that of the sun. It is, however, found by experience that, although easy to think of this tangency, it is difficult to observe it with the requisite precision, owing to the imperfection of vision, and especially to the irradiation produced by the earth's atmosphere and by any imperfections in the telescope. The phenomena to be really observed are defined as follows in the instructions of the International Conference held at Paris in 1881:

"At ingress the moment to be noted is that at which the observer sees for the last time an evident and persistent discontinuity in the apparent limb of the sun near the point of contact with Venus."

"At egress the moment of the first appearance of a well-marked and persistent discontinuity in the illumination of the apparent limb of the sun at the point of contact."

However well these definitions may apply to the actual phenomena, they are not sufficient, without further explanation, to enable the observer to know what is contact

under all circumstances. Indeed, the Conference itself added a number of instructions of what was to be looked for under special circumstances. For these instructions the following are, however, substituted by the American Commission.

We must first remember that just before internal contact at ingress the sun's limb will be broken off by the advancing planet, and that portion which is visible near the point of contact will present the appearance of two fine sharp horns, the points of which will slowly approach each other. The moment of true internal contact is evidently that at which these points exactly meet. But since they cannot be seen by the eye to meet until the completed line of light becomes thick enough to be seen, the observer must not expect to see the thread of light actually complete until after the contact has passed. As a general rule, therefore, he must note what is to be seen just before this thread of light becomes evidently complete. What he sees will depend very largely upon the clearness and steadiness of the air. The most favorable circumstances for observing true contact are those in which the cusps appear steady and sharply defined against the black background of the sky. There will then be little difficulty in catching the moment at which they are first about to meet, which will be that of true contact.

In most cases, however, especially if the sun is low, the outline of the cusps will be wavy and serrated, their ends will be more or less rounded instead of being sharp, and their outline will be continually changing in consequence of the apparent undulating motion produced by the atmosphere. The greater this vibratory motion and the more the cusps are blunted the more difficult it will be to catch the moment of true contact. The following rules are then to be borne in mind by the observer. So long as the dark region between the cusps which connects the black disk of Venus with the black sky outside the sun retains its full darkness, without any apparent motion or undulation going across it, so long contact has not occurred, and this although the planet may seem entirely within the true outline of the sun. It would be well for the observer to have an assistant at the chronometer to whom he can from time to time call out the words "not yet." The assistant should write down the second by the watch or chronometer at which the observer commenced to pronounce these words.

Instead of the cusps uniting into a fine, steady line of light, the observer may at a certain moment begin to see an undulating motion extending all the way across this dark space. He will soon after see that this motion is due to the continually increasing line of light, which is broken into threads and waves by atmospheric undulation. From and after the time that this undulation is permanently seen contact is certainly passed. It would be well, when it is first fully recognized, that the observer should call out "past" to his assistant, who should note the time at which the word is spoken. If he has made no mistake in his estimates the time of contact will be limited between the last moment at which "not yet" was spoken and the first moment at which "past" was pronounced.

In the event of the cusps appearing much rounded, the further Venus appears inside the disk of the sun, as completed in the imagination by continuing its outline across the dark region, the more careful must the observer be to catch the first line of true sunlight extending across. It may be assumed that if the seeing is at all good the undulating light of the sun's limb will be clearly recognized in a very few seconds after

the true time of contact. On the other hand, he must be on his guard against mistaking some slight haziness around the point of contact for the appearance of true sunlight. It is also possible, in case of a very bright but undulating image, that the sharp cusps may from time to time be momentarily brought together by atmospheric undulations before contact really occurs. These are points upon which the observer must be left to his own judgment. He must in all cases try to estimate what the phenomenon would be if there were no undulations, and he will be assisted in this by taking particular note of the appearance at those moments of steadiness which generally occur every few seconds in the worst atmosphere. The trouble to which observers are prone is that of catching some phenomenon or undulation, which really occurs only from time to time, and fixing the attention on it as though it were permanent. It is so easy to imagine that one sees irregular phenomena that the observer must be especially careful to distinguish what is permanent from what is an accidental product of atmospheric vibration.

The preceding directions apply principally to those cases in which the air is clear and the sky blue If the observation is made through a sky so covered by clouds or haze that there is no striking contrast between the brilliancy of the sun and that of the surrounding sky, the observation may be extremely difficult, because the completion of the thread of light will probably not be seen until a considerable period after actual contact. It is therefore best in such cases that the observer should note the last moment at which he felt sure the limbs did not become tangent and the first moment at which it became permanently evident that the planet had passed entirely within the sun. Perhaps no better definition can be given of contact under such circumstances than that it is the moment when the limbs are really tangent.

Whatever moment the observer may note, it is indispensable that he give an accurate statement of the appearance presented by the sun and planet at that moment, accompanied by a drawing if necessary. If he is able also to give a drawing or description for the moment at which he last spoke the words "not yet," and at which he first said "past," it will be well to do so. At the same time the useless multiplication of times is to be guarded against, owing to the distraction thus produced.

Egress.—At egress the phases occur in inverse order, so that the same directions will apply when properly interpreted with the respect to time. The following points are, however, to be especially noted:

As the thread of light between the limbs of the sun and planet becomes very thin it will probably appear to darken, partly from atmospheric irradiation and partly from the eye being less affected by a thin line than by a broad band of equal brilliancy. If the atmosphere is undulating this thread may be expected to break up into a mass of undulating threads of light, continually changing their form and appearance. So long as this undulating mass continues to be seen in motion all the way across the dark space near the point of contact, so long contact has not occurred. If, however, the appearance of congealing into a hard immovable mass is presented, the moment of such seeming congelation is that of true contact.

In looking for the complete disappearance of the undulating light the observer must be on his guard against mistaking the illumination of the outline of Venus, produced by the atmosphere of the planet, for the true light of the sun's limb. There were supposed to be some cases during the transit of 1874 in which the observer,

watching for the fading line of light to disappear, found himself really observing this atmospheric outline after contact was past This is a point on which, in the absence of complete experience, no definite instruction can be given to the observer, and he must rely upon his own judgment to guard against a mistake of this kind

In the case of parties supplied with double-image micrometers it is recommended that measures of the thickness of the band of light between the two limbs be commenced as soon after internal contact as the observer has made all his necessary records and notes It will also be well, twenty minutes before second internal contact, to commence similar measurements of the thickness of the point of light between the limbs Great care must, however, be taken to stop these measures and replace the micrometer by the eye-piece in good time to make a careful observation of contact

Last contact—The observer should note the last moment at which he certainly and distinctly saw the vanishing-notch made by the receding planet To assist in this it will be well to pronounce the word "notch" from time to time and have the times noted by the assistant

XXXVII —METHODS OF RECORDING CONTACTS.

To make the best possible observations of contacts the observer must be well prepared to note the times of such phenomena as he may see, and this without any liability to errors of 10, 20, or 30 seconds, or a whole minute If he has to look at and read a time-piece himself there is danger of such errors They may be avoided by employing a chronograph, but in observing contacts there are two difficulties connected with the use of this instrument The first is that in the event of other signals than those of contact being made, whether by accident or design, it may be difficult to recognize the meaning of the several signals The second is that in general the instant of contact can be recognized only by watching the course of the phenomena both before and after that event, and thus the observer is not ready to record the contact till some seconds after it has occurred Still, with proper precautions against these difficulties, a chronograph may be used

Experience has shown that when an observer notes the times himself, the surest way of guarding against errors in the seconds is to have an assistant at the clock or chronometer to beat every second with a key, or small hammer, upon a board By this plan a pocket watch may be used in the absence of a better time-piece At the moment of each beat the assistant must call out only the units or the tens of the second Thus, beginning at ten seconds, the calls will be ten, one, two, three, etc, twenty, one, two, etc, thirty, one, two, etc The reason for not calling the numbers in full, twenty-one, twenty-two, etc, is that their distinct pronunciation would require such a considerable portion of a second that the observer might be in doubt which beat any one number belonged to The simple numerals from one to ten may be pronounced simultaneously with the beats, so as to leave no doubt An assistant beating in this way may give time to several persons

If the observer employs an assistant at the time-piece to read off and record his times, he must also arrange beforehand a system for registering notes respecting the phenomena Such notes will be "not yet," "haziness near the point of contact,"

"flashes around the planet before contact," "atmosphere of Venus clearly illuminated," etc. If several such notes have to be made, two assistants will be necessary—one to write them down and the other to record the times. To co-ordinate the notes with the times, the letters A, B, C, etc., may be employed. The assistant at the chronometer is then to record the chronometer time at which the letter was spoken, and opposite it the letter itself, while the other assistant is to write down first the letter and then the note.

XXXVIII.—DIAMETER OF VENUS

If the chief of party can spare any time from the photographic operations between second and third contact, it should be employed in measuring the diameter of Venus with the double-image micrometer. Such measures may be made in groups of sixteen, viz. Four measures of the polar diameter, two of them being made with the index to the right of zero, and two with it to the left, eight measures of the equatorial diameter, four being with index to right, and four with index to left, and lastly, four more measures of the polar diameter, two being with index to right, and two with index to left. By this arrangement the zero point and all errors symmetrical with the time are eliminated. FORM II contains some observations of the diameter of Mercury, made during its transit in May, 1878, at Austin, Texas, which are given as a specimen of such work. The numbers in the first column are, the chronometer time when the measurements were begun, namely 9 14 = $9^h\ 14^m$, the reading of an aneroid barometer, 28 98 inches, the temperature of the air, 92° Fahrenheit, and the chronometer time when the measures were finished, 9 30 = $9^h\ 30^m$. The other columns are sufficiently explained by their headings. One revolution of the micrometer-screw is equivalent to 17″ 208, and the diameters are the product of the sums by one-quarter of this value.

FORM II

Time, Barom Ther	Diameter Measured	Readings of Screw		Differences	
		To Left	To Right	Polar	Equatorial
		r	r	r	r
9 14	Polar	14 91	13 51	1 40	
		91	51	1 40	
	Equatorial	14 91	13 51		1 40
28 98		92	54		38
92 0		93	53		40
		92	52		1 40
9 30	Polar	14 91	13 51	1 40	
		91	52	1 39	
Sums				5 59	5 58
Diameter of Planet				24″ 05	24″ 00

XXXIX.—DATA REQUIRED.

For convenience of reference, the data which will be required in reducing the observations are here enumerated. D 3, 4, 5, 6, 7, G 3, 6, 8, 9, 10, H 3, and K 1 can be best determined in Washington. All the others must be determined in the field, and it will be the special duty of the chief of party to see that nothing needful is omitted from the record.

A.—Name of station.

B.—Latitude and longitude of station.

C.—With every observation, the name of each person employed in making it, and the part he took in the work, must be recorded.

D.—For the meridian instrument:

1. Maker's name and number.
2. Size and power of telescope.
3. Correction for flexure.
4. Correction for inequality of pivots.
5. Value of scale of striding level.
6. Value of scale of zenith distance level.
7. Value of one revolution of the micrometer screw.
8. Intervals of transit wires.
9. Intervals of micrometer wires.

E.—For the chronometers:

1. Makers' names, numbers, and descriptions, whether mean time or sidereal, number of hours on dial, break-circuit or not, etc.
2. The correction of each chronometer to local time whenever it is used.

F.—The maker's name, and number of the chronograph.

G.—For the photoheliograph:

1. Number of objective.
2. Number of measuring-rod.
3. Length of measuring-rod at some definite temperature.
4. Temperature of measuring-rod whenever it is used.
5. Number of jaw-micrometer.
6. Correction to be applied to readings of jaw-micrometer.
7. Distance from back surface of objective to front surface of reticule-plate.
8. Distance from back surface of the objective to its second principal point.
9. Thickness of reticule-plate.
10. Refractive index of reticule-plate.
11. Interval between reticule-plate and collodion-film.
12. Azimuth of the center of the reticule-plate.
13. Level, or zenith distance, of the center of the reticule-plate.
14. When the instrument is in use, the temperature of the atmosphere in the shade, the temperature in the photographic house, and the reading of the barometer and its attached thermometer, must be recorded every half hour.

G.—For the photoheliograph—Continued.

15. Every negative must have its number marked upon it by a diamond. With each negative must be recorded the chronometer time of its exposure, a sidereal chronometer being used; the direction (east or west) of the small arm on top of the frame, from the center of which the plumb-line is suspended; and the direction of motion of the exposing-slide (east or west).

H.—For the engineer's level, or level of precision:

1. Maker's name and number.
2. Size and power of telescope.
3. Value of level-scale.
4. Value of micrometer-screw.

I.—For the theodolite:

1. Maker's name and number.
2. Size and power of telescope.
3. Diameters of limbs, and least reading.

K.—For the equatorial telescope:

1. Maker's name and number
2. Size of telescope.
3. Powers of eye-pieces.
4. Value of one revolution of the screw of the double-image micrometer for each of the two front lenses.
5. With every observation, the power employed in making it must be recorded.

The utmost care must be taken to have accurate knowledge of the local time on the day of the transit. Owing to the uncertainty of weather, no fair night must be allowed to pass during the week preceding December 6 without the observation of star-transits for time and azimuth; but if clouds prevent such observations, then, if possible, the transit of both limbs of the sun must be observed daily during the same period, the telescope being reversed between the limbs. On the day of the transit all the chronometers at the station must be compared in the morning, and again in the evening.

On December 5th all the apparatus must be inspected to make sure that it is in good working order, and care must be taken that the adjustments 4 and 5 of section V are correct. Both on the day of the transit and on the day preceding, G 7 and G 13 must be very accurately determined; and G 12 must be deduced from the transits of stars observed on nights before and after the transit of Venus, but as near to that event as possible.

Examples showing how observations of G 13 should be made and recorded are appended. In explanation of them it is only necessary to say that Form III refers to observations with an engineer's level, and Form IV to observations with a level of precision. The theory of these observations has been already given in section V. That end of the bubble which gives the largest reading is toward the high end of the line, and the amount of elevation is found by multiplying the number in the line "Difference" into one-sixteenth of the value of one division of the level scale. For the level Stackpole and Brother, No. 1510, the valve of one division was $6''.54$. One-sixteenth of this is $0''.409$, which multiplied by 9.8 divisions gives $4''.01$. For the level

Stackpole and Brother, No 1489, the value of one division is 1″ 74, one-sixteenth of this is 0″ 109, which multiplied by 22 0 divisions gives 2″ 40 The large collimation error of the telescope of this level is noticeable

FORM III —*Observations made with the engineer's level* STACKPOLE *and* BROTHER, *No* 1510, *to determine the inclination of the line of collimation of the horizontal photoheliograph at Hobart Town, Tasmania, December* 9, 1874

Object Observed	End of Bubble			
	North	South	North	South
	d.	d	d	d
Reticule	45 0	13 9		
	44 6	13 9		
Transit			12 0	43 0
			12 5	43 4
			12 2	43 3
			12 2	43 0
Reticule	44 0	13 0		
	44 0	13 0		
Sums	177 6	53 8	48 9	172 7
Sums	231 4		221 6	
Difference	9 8			
North end high, 4″ 01				

FORM IV —*Observations made with the level of precision*, STACKPOLE *and* BROTHER, *No* 1489, *to determine the inclination of the line of collimation of the horizontal photoheliograph at Washington, D C, September* 9, 1882

Focusing Pinion	End of Bubble	Level Direct	Level Reversed
Right	N	23 5	44 0
	S	51 5	15 0
Left	N	13 0	54 0
	S	42 0	25 5
	N	12 0	55 0
	S	41 0	26 0
Right	N	23 0	44 5
	S	51 5	15 5
Sums		257 5	279 5
Difference		22 0	
North end high, 2″ 40			

XL.—RECORDS OF OBSERVATIONS AND OPERATIONS.

Of the journals and memorandum-books, each observing-room is to have at least one for its exclusive use, in which every operation must be recorded in detail, with all the particulars necessary to its being fully understood. A journal is also to be kept, in which all the operations of the party must be entered.

In addition to the original record of the observations, a duplicate record must be made with ink, at the earliest possible moment. The following rules respecting the correction of supposed mistakes must be attended to in the case of each set of records:

Original rough record.—In this record a number once written should never be erased. If the observer detects a wrong number immediately after writing it, he must draw a line through it and write the correct number beside it. If it is concluded from subsequent observations that a number is probably wrong, that fact must be noted, and the correct number indicated; but the record as written must not be altered. Numbers should not be inserted in this record which are the result of calculation. For instance, if the observer fails to note the minutes of the chronometer corresponding to any observation, he must not conclude what they were from the preceding or subsequent observations, and then put them in, but must omit them entirely, unless such omission would cause subsequent uncertainty. In that case the necessary numbers may be inserted, provided it is done in such a manner as to show that they were not directly observed, but are concluded from other parts of the record. To indicate this, a circle may be drawn around them.

Should it be found necessary from any cause whatever to make the first rough notes of an observation upon loose slips of paper, these slips must be carefully pasted into the proper note book, in immediate proximity to the formal record which has been copied from them.

The duplicate or fair copy.—The second record is to be copied from the first as soon as practicable after making the observations, so that if mistakes exist they may be detected and corrected. If blank forms for the observations are provided, they may be used for the second copy as well as for the first; but everything for which they are not available must be copied in chronological order into a single book. In this duplicate copy greater liberty will be allowed respecting additions and alterations than in the original, the object being to make a complete and correct record; but in the event of numbers being added as the result of calculation, they should be underscored with red ink, or otherwise indicated.

All records of observations must include every particular necessary to their being completely understood. For instance, wherever time is given, the particular timepiece employed must be designated; where level readings are given, the direction of each end of the level, east, west, north, or south, must be recorded; when the images of lines on the plate-holder are observed with the transit, the direction of the image from the middle wire, whether right, left, north, or south, must be stated, as well as the direction of the eye-piece of the instrument, east or west; and all photographs of the sun must indicate the exact time at which they were taken, and must be so marked that the position of the plate when in the holder—that is, the top, bottom, east and west sides—can always be distinguished.

In using the chronograph, the minutes and seconds must always be marked upon the sheet at least twice during each series of observations. All chronograph sheets must be preserved and sent home with the other original records, but in addition to this they must also be read off and recorded in the proper books.

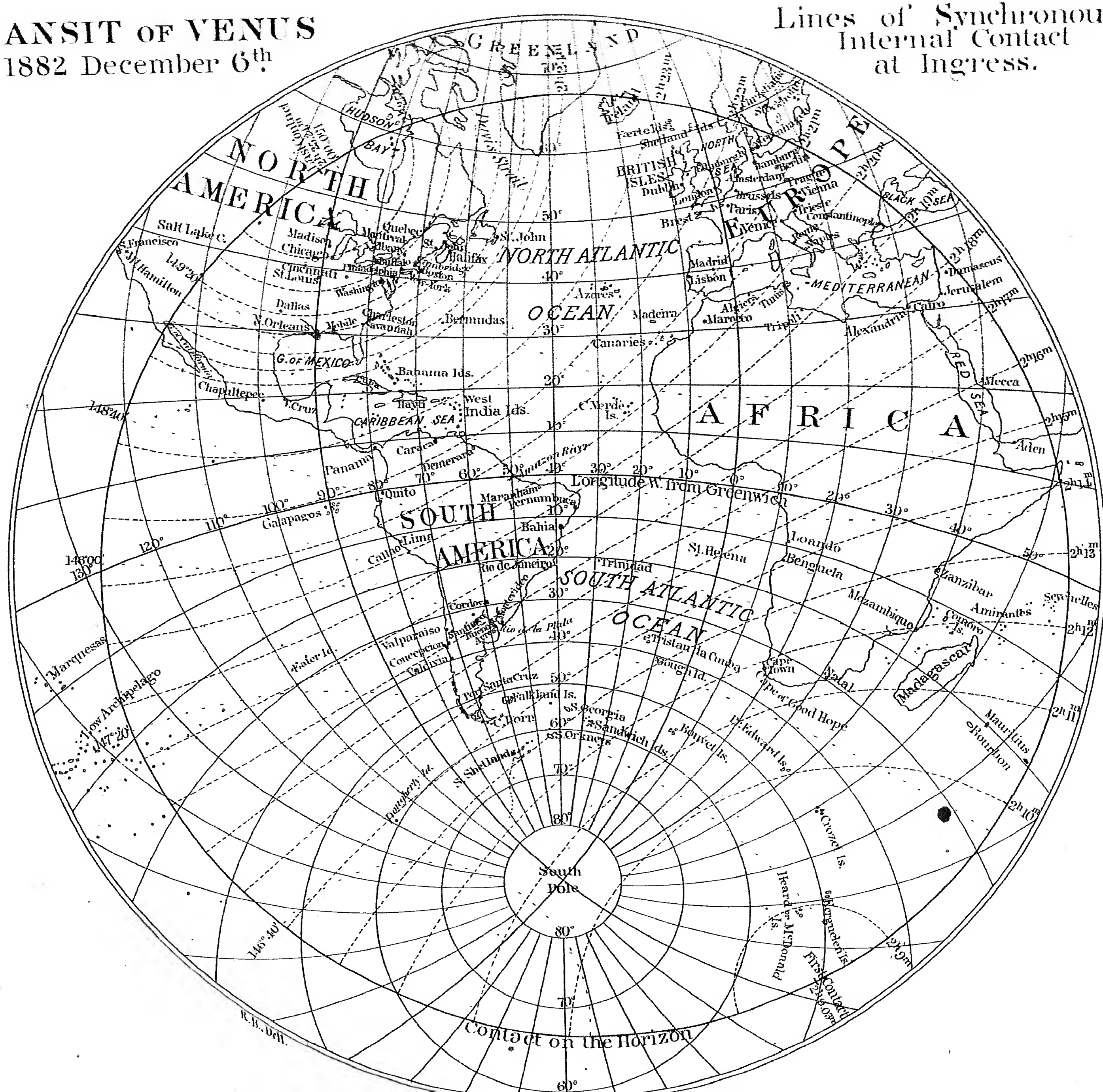
XLI.—TRANSMISSION AND PUBLICATION OF OBSERVATIONS.

All members of the parties are prohibited from publishing their observations, or results, without authority of the commission, but this prohibition is not intended to prevent any general statements respecting the nature and success of the work which the observers may choose to make. In cases of co-operation with any other individual or party, the chief of the party is authorized to communicate to the other copies of all observations necessary to the special end for which the co-operation was entered into.

The chief of each party will transmit all the records to the president of the commission as soon as practicable after the completion of the observations. They will be sent in separate packets, one containing the journal and all the original pencil memoranda of the observations, the other the fair copy already directed to be made. If practicable, the two packets must be sent at different times and by different conveyances. From ports where an American consul is stationed, they may be forwarded by him through the Department of State, in which case he must be notified to send the two packets by different ships.

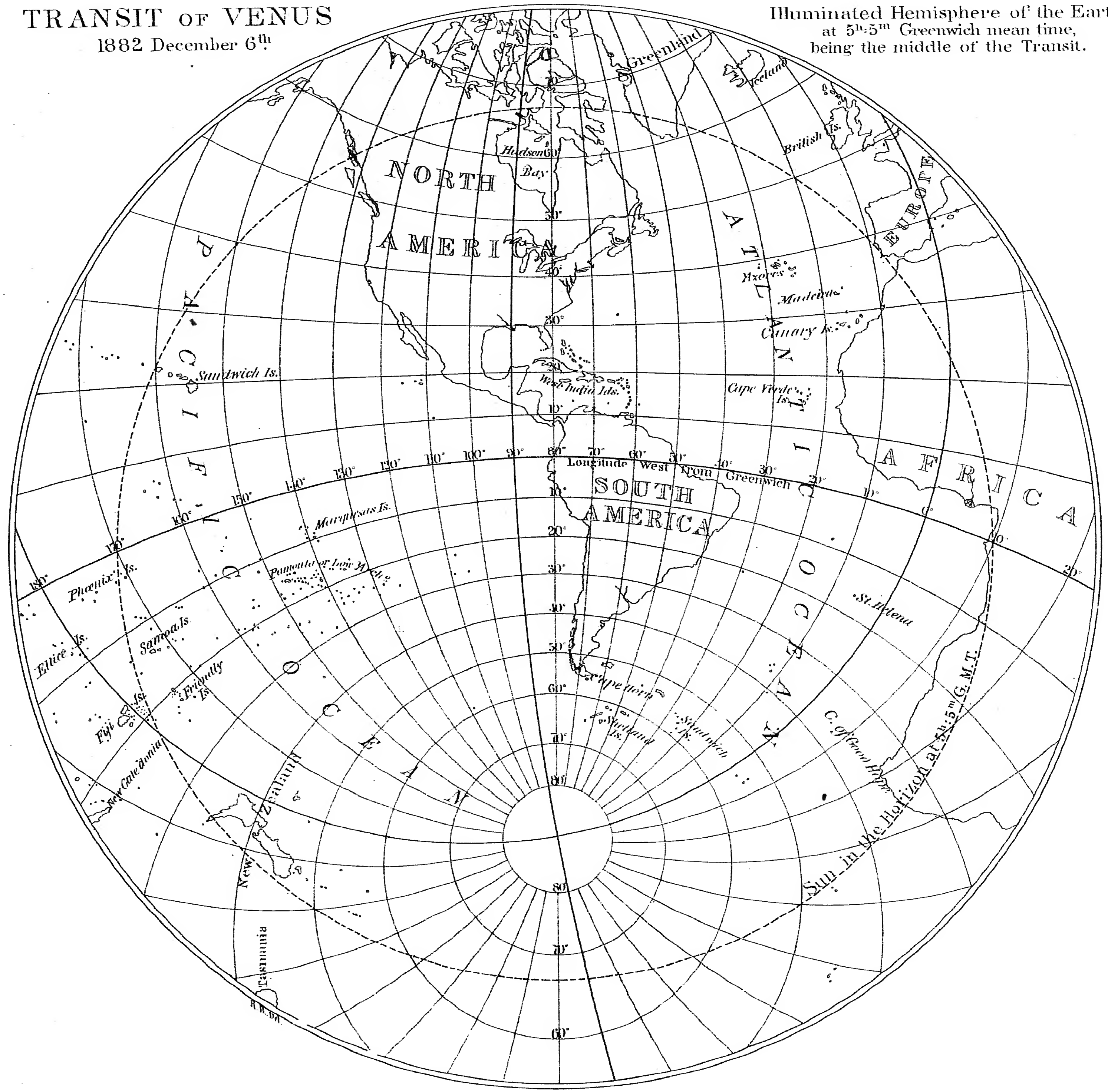
TRANSIT OF VENUS
1882 December 6th.

Lines of Synchronous
Internal Contact
at Ingress.



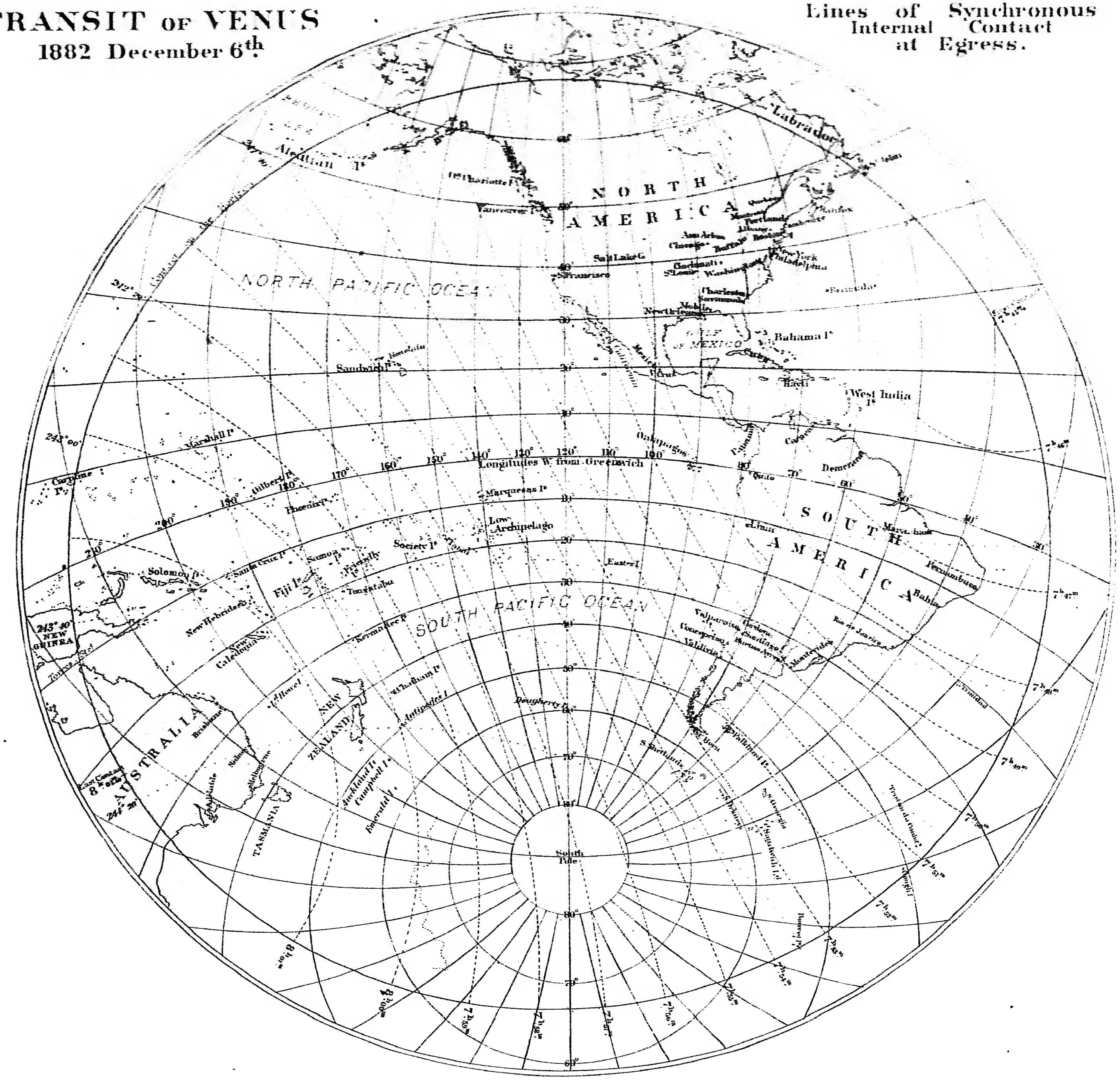
TRANSIT OF VENUS
1882 December 6th.

Illuminated Hemisphere of the Earth at 5h 5m Greenwich mean time, being the middle of the Transit.



TRANSIT OF VENUS
1882 December 6th.

Lines of Synchronous Internal Contact at Egress.



Scale for Altitudes
30° 25° 20° 15° 10° 5° 0°

TRANSIT OF VENUS

1882 December 6th

Paths described by several Stations through diurnal motion as seen from the Sun during the Transit.